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Accurate numerical methods for the collisional motion of (heated) granular flows

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Abstract

In this paper, we extend the spectral method developed in [L. Pareschi, B. Perthame, A Fourier spectral method for homogeneous Boltzmann equations, Trans. Theo. Stat. Phys. 25 (1996) 369–383; L. Pareschi, G. Russo, Numerical solution of the Boltzmann equation I: Spectrally accurate approximation of the collision operator, SIAM J. Numer. Anal. 37 (2000) 1217–1245] to the case of the inelastic Boltzmann equation describing the collisional motion of a granular gas with and without a heating source. The schemes are based on a Fourier representation of the equation in the velocity space and provide a very accurate description of the time evolution of the distribution function. Several numerical results in dimension one to three show the efficiency and accuracy of the proposed algorithms. Some mathematical and physical conjectures are also addressed with the aid of the numerical simulations.

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Keywords: Inelastic Boltzmann equation; Spectral methods; Granular gases; Homogeneous cooling

1. Introduction

In kinetic theory, granular fluids far from equilibrium are usually modelled by inelastic hard spheres describing dissipative short range interactions between molecules. The interest in granular matter has strongly stimulated new developments in kinetic theory of granular gases.

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A granular gas can be viewed as a set of large macro-particles with short range repulsive core interactions, in which energy is lost in the inelastic collisions. These macro-particles are described by a distribution function $f(t, x, v)$, which depends on time $t \geqslant 0$, position $x \in \mathbb{R}^d$ and velocity $v \in \mathbb{R}^d$, $d \geqslant 1$, and solves a Boltzmann type equation [15–17]

$$\frac{\partial f}{\partial t} + v \cdot \nabla_x f = Q(f, f). \tag{1}$$

The collision operator $Q(f, f)$ describes the binary collisions, which only conserve mass and momentum since energy is dissipating. The inelastic collisions are characterized by a restitution coefficient e $(0 < e < 1)$, where $(1 - e^2)$ measures the degree of inelasticity.

Granular gases reveal a rich variety of self-organized structures such as large scale clusters, vertex fields, characteristic shock waves and others, which are still far from being completely understood. Applications of such systems range from astrophysics (stellar clouds, planetary rings), to industrial processes (handling of pharmaceuticals) and environment (pollution, erosion processes). Despite their importance in applications, deterministic numerical studies involving the full three-dimensional Boltzmann or Enskog dissipative kinetic equations have never been addressed before.

As a first step towards the numerical solution to the full problem, in this paper we will focus on the time evolution and the steady states of self-similar solutions to (1) in the spatially homogeneous case. There are several reasons behind this choice. First of all, the numerical study of the homogeneous cooling process is of major importance to understand the physics of such systems and for the construction of suitable equations of hydrodynamics. Non-Maxwellian equilibrium states, finite time energy extinction and quasi-elastic asymptotics [36,28,4] are just some of the non-trivial homogeneous behaviors. Not to mention the fact that most of the numerical difficulties related to the solution of (1) are due to the presence of $Q(f, f)$ and not to the transport part or the additional heating source.

Second, from a theoretical point of view, the study of the large-time behavior of the solution to the spatially homogeneous Boltzmann equation received a lot of interest in recent years, and essential progresses have been made in particular on the Boltzmann equation for inelastic Maxwell particles, both for the free case without energy input [4,10], and for the driven case [13,14,5].

It is remarkable that, on the contrary to elastic collisions, partially inelastic collisions have a non-trivial outcome as well in one dimension, and the one-dimensional idealization is a non-trivial adjunct to more realistic studies. One-dimensional Maxwellian inelastic gases where studied in [1]. This study led to the discovery of an exact similarity solution for a freely cooling Maxwellian inelastic gas [1] (which corresponds to the well known "BKW" solution [3,18] since they are identical in the Fourier space as explained in [5]). This solution, which has an algebraic high energy tail like $1/v^4$, can be used to test the class of initial values that are attracted in large-time. A different one-dimensional kinetic equation, which can be considered as a dissipative version of Kac's model, have been recently introduced in [32] to fully understand, at least in simplified models, the importance of the amount of dissipation in the cooling problem.

Real models, in which particles undergo binary hard-sphere interactions, or the coefficient of restitution depends on the relative velocity, have been less studied. The behavior of a hard-sphere granular gas in presence of some additional external source of energy in the system (a heat operator), has been recently investigated in [24], and the existence of non-trivial stationary states has been found.

Here, we will extend the spectral method recently presented in [30,31] for the classical Boltzmann equation to the inelastic situation. At variance to Monte Carlo methods the spectral method has shown to be extremely accurate and thus very suitable to test mathematical and physical conjectures. We refer the reader to [17,29,22] for a detailed discussion on spectral methods for the Boltzmann equation and their application to non-homogeneous situations. Finally, we mention here some recent works where the numerical solution of some kinetic model for granular gases has been considered [28,25].