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Efficient parallel implementation of the nonparaxial beam propagation method

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ABSTRACT

An efficient parallel implementation of a nonparaxial beam propagation method for the numerical study of the nonlinear Helmholtz equation is presented. Our solution focuses on minimizing communication and computational demands of the method which are dependent on a nonparaxiality parameter. Performance tests carried out on different types of parallel systems behave according theoretical predictions and show that our proposal exhibits a better behavior than those solutions based on the use of conventional parallel fast Fourier transform implementations. The application of our design is illustrated in a particularly demanding scenario: the study of dark solitons at interfaces separating two defocusing Kerr media, where it is shown to play a key role.

1. Introduction

Nonlinear optics is one of many scientific fields that have experienced a great development due to the progress in computational science. Numerics are essential to provide information in those nonlinear scenarios where analytical solutions do not exist or are difficult to obtain. Such is the case found in the analysis of the nonlinear propagation of optical wave-packets localized either in space or time, i.e. continuous-wave (CW) beams [1] and optical pulses in fibers [2], respectively, where the nonlinear Schrödinger (NLS) equation has been widely used. The NLS equation belongs to a class of partial differential equations [3] whose full numerical integration can be performed using the split-step Fourier (SSF) method [4]. Nevertheless, the slowly varying envelope approximation (SVEA) assumed in the derivation of the NLS equation is not valid when addressing the nonlinear propagation of optical signals in many different scenarios.

The SVEA, when applied to CW beams, is also known as the paraxial approximation. Nevertheless, nonparaxiality can easily become a misunderstood term because it applies not only to the evolution of ultra-narrow high-intense beams, but also to the propagation of broad beams of relatively low intensity at *large angles* in relation to a longitudinal axis. The first source of nonparaxiality was questioned by Akhmediev and Soto-Crespo [5], who unveiled the limitations of the scalar NLS equation in scenarios of strong focusing. Full vectorial analysis were thus proposed to address the evolution of both bright [6] and dark [7] nonparaxial solitons in nonlinear media. Unlike this, the second type of nonparaxiality can be described by means of a scalar field equation, such as the nonlinear Helmholtz (NLH) equation [8,9]. Exact analytical solutions of the NLH equation have been provided in the framework of the Helmholtz theory [9–11] and substantial differences in relation to previous

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paraxial analyses [9–15] also manifest in those nonlinear scenarios where it is essential to preserve the angular character of the problem. The study of bright soliton collisions [16] or the interaction of Kerr solitons with nonlinear interfaces [17–19] represent two such examples.

Focusing our interest on the NLH, numerical methods have also been developed to investigate nonlinear propagation phenomena associated to this equation where backscattered waves appear accompanying the propagation of a forward propagating beam. The elimination of backward waves has concentrated much of the attention of numerical methods such as the two-way arbitrary boundary conditions model [20,21] or the nonparaxial beam propagation method (NBPM) [22]. The validity of the former has been demonstrated in the arrest of soliton collapse for the (2 + 1)-D NLH equation or in the formation of nonparaxial solitons for the (1 + 1)-D NLH equation [23]. The latter, has been essential in the development of the Helmholtz theory [9,11–15] and its efficient parallelization constitutes the main object of this paper.

The NBPM [22] is obtained by approximating the evolution equation for the field envelope of a solution of the NLH equation with a difference–differential model. While the longitudinal evolution is computed according to a finite difference scheme, the term associated with linear diffraction is evaluated in the spectral domain thus demanding the application of successive forward and backward fast Fourier transforms (FFT) [24,25] which constitute the computational core of the NBPM. As regards parallel computing, the NBPM is an excellent candidate to be parallelized. The core of the implementation is the paradigm of a Single Instruction Multiple Data (SIMD) problem [26,27] where massive additions and multiplications associated to the computation of the FFTs must be applied to a regular data set. Therefore, the use of parallel FFT implementations based on either the binary exchange method [28,29] or the transpose method [30–32] can be considered as a straightforward approach for the parallelization of the NBPM. Such is the case, for instance, of the recent Fast Fourier Transform in the West (FFTW) [33] whose parallel implementation has already been used in the core of the SSF for studying coupled NLS equations [34] or the Korteweg–de-Vries (KdV) equation [35]. Nevertheless, as we show in this work, much more efficient schemes are possible for the parallelization of the NBPM whenever one exploits the specific features of the numerical method in order to minimize both communication bandwidth and computational costs in relation to state-of-the art routines. Such is the case of the parallel implementation of the NBPM we present in this work, denoted as PNBPM. Performance results carried out on several parallel systems confirm that the PNBPM reduces the overall execution time involved in the NBPM when compared with the use of state-of-the-art routines in the core of the NBPM.

This paper is structured as follows. In Section 2, we briefly review the NBPM and highlight those issues which are essential for the parallel implementation of the method. Different approaches proposed as candidates to constitute the parallel implementation of the NBPM core are discussed in Section 3. Section 4 presents the main contribution of this work, which is the PNBPM. Both computational and communication costs are separately analyzed and demonstrated to depend on the nonparaxiality parameter present in the NLH. Section 5 is devoted to show the results of performance tests carried out on different parallel systems when our approach is compared to different alternatives. Section 6 illustrates the essential role played by the PNBPM proposed in this paper in the study of a problem whose numerical solution is particularly involved, namely the evolution of black Helmholtz solitons at the interface separating two defocusing Kerr media. Finally, in Section 7 we summarize the conclusions of the work.

2. Nonparaxial beam propagation method

The NBPM permits the numerical integration of the evolution equation

$$\kappa \frac{\partial^2 u}{\partial \zeta^2} + i\frac{\partial u}{\partial \zeta} + \frac{1}{2}\frac{\partial^2 u}{\partial \xi^2} + |u|^2 u = 0 \tag{1}$$

used in the analyses of Helmholtz solitons in Kerr focusing media [9–11]. u represents the complex envelope of a forward propagating electric field $E(x,z) = (n_0/n_2 k L_D)^{1/2} u(x,z) exp(ikz)$ where n_0 and n_2 account for the linear and nonlinear refractive indexes of the Kerr-type medium $n(E) = n_0 + n_2|E|^2$, respectively. In Eq. (1) $u(\xi,\zeta)$ is expressed in terms of the normalized transverse and longitudinal coordinates $\xi = 2^{1/2}x/w_0$ and $\zeta = z/L_D$, respectively. w_0 is a transverse scale parameter, equal to the waist of a reference Gaussian beam of diffraction length $L_D = kw_0^2/2$. $\kappa = 1/(kw_0)^2$ is a nonparaxiality parameter, relating beam width and wavelength $k = 2\pi/\lambda$. Eq. (1) is directly obtained from the NLH equation with no approximation [9] and, thus, the results obtained from both equations are fully equivalent. Both the model equation shown in Eq. (1) and the NPBM algorithm have been extended to address the propagation of dark solitons in defocusing Kerr media, bistable solitons in cubic-quintic nonlinear materials, or inhomogeneous nonlinear media with planar boundaries [12,15,17–19].

A comparison of Eq. (1) with the NLS equation

$$i\frac{\partial u}{\partial \zeta} + \frac{1}{2}\frac{\partial^2 u}{\partial \xi^2} + |u|^2 u = 0 \tag{2}$$

reveals that the term associated to the nonparaxial parameter is missing in the NLS equation. Such omission accounts for the paraxial approximation (or, equivalently, the SVEA) and discards the rapid evolution of the complex field envelope along the ζ coordinate. Besides its physical relevance in the context of Helmholtz solitons, κ is also going to play a fundamental role in the parallelization of the NBPM, as it will be shown in Section 4.