Journal of Visual Languages and Computing 25 (2014) 935–944

Contents lists available at ScienceDirect

Journal of Visual Languages and Computing

journal homepage: www.elsevier.com/locate/jvlc

A normal form for spider diagrams of order

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ARTICLE INFO

Article history:
Received 25 September 2014
Received in revised form
24 October 2014
Accepted 28 October 2014
This paper has been recommended for acceptance by S.-K. Chang
Available online 31 October 2014

Keywords:
Spider diagrams
Logic
Normal form
Euler diagrams

ABSTRACT

We develop a reasoning system for an Euler diagram based visual logic, called spider diagrams of order. We define a normal form for spider diagrams of order and provide an algorithm, based on the reasoning system, for producing diagrams in our normal form. Normal forms for visual logics have been shown to assist in proving completeness of associated reasoning systems. We wish to use the reasoning system to allow future direct comparison of spider diagrams of order and linear temporal logic.

1. Introduction

Shin's rebirth of Peirce's α and β systems for reasoning [15] has produced a variety of Euler diagram based visual logics, for example [7,4,17,18]. Euler diagram based visual logics allow reasoning about sets, their elements and their relationships. Associated with visual logics are reasoning systems that embody equivalence between diagrams [2,10,19]. Spider Diagrams of Order (SDoO) and Second-Order Spider Diagrams [3] differ from the main body of work on Euler diagram based logics as elements of their token syntax were designed to be as expressive as star-free regular languages and regular languages respectively.

Weakly expressive language classes, such as regular languages and star-free regular languages, are used to formalise real-world temporal specifications [5,12]. Due to the real-world application there has been recent interest in incorporating temporal semantics in these diagrammatic logics [1,14]. In this paper we address the problem of adding temporal semantics to Euler diagrams by adding a syntax and semantics for specifying order of the elements. Furthermore, we develop the first reasoning system for an Euler diagram based logic that includes an order relation. In demonstrating our reasoning system for spider diagrams of order we produce both a normal form and an algorithm to produce the normal form. Our algorithm also contributes to the recent interest in normal forms for Euler diagram based logics [9].

In Section 2 we define the syntax and semantics of spider diagrams of order. In Section 3 we present each of our reasoning rules. Thereafter, in Section 4 we present our normalisation algorithm by example. An implementation of our algorithm is available under an open-source license at https://github.com/AidanDelaney/SpiderReasoning.

2. Spider diagrams of order

The Euler diagram in Fig. 1(a) contains three labelled contours and six zones. A *zone* is defined to be a pair, (*in*, *out*), of disjoint subsets of the set of contour labels. The set *in* contains the labels of the contours that the zone is inside whereas *out* contains the labels of the contours that the zone is outside. The set of all zones is denoted $\mathcal{Z}$.

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http://dx.doi.org/10.1016/j.jvlc.2014.10.029
1045-926X/

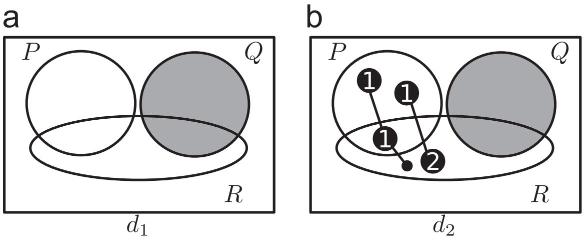

Fig. 1. An Euler and a spider diagram.

A *region* is a set of zones. As an example, there exists a zone inside the contour P but outside both contours Q and R denoted $(\{P\},\{Q,R\})$. The zone inside the bounding box and outside all contours can be described by being inside $\varnothing$ and outside $\{P,Q,R\}$. We note that there is no zone in the diagram corresponding to $(\{P,Q\},\{R\})$ i.e. there is no zone inside contours labelled P and Q but outside the contour labelled R. Euler diagrams may be conjoined using the symbol $\wedge$, disjoined using the symbol $\vee$ or negated using the symbol $\neg$.

A *Spider Diagram of Order* is an Euler diagram containing one or more graphs. The vertices of a graph are labelled with '$\bullet$' or an integer. A graph is restricted such that it is acyclic and may not have more than one vertex of a given label in a given zone. To maintain consistency with the literature we call graphs of this form *spiders* and term a vertex within a graph to be a *spider foot*. The diagram in Fig. 1(b) contains two spiders, one spider consisting of three feet labelled '1', '1' and '$\bullet$', and the other spider contains two feet labelled '1' and '2'. In the following definition, as throughout the paper, we use $\cup$ to mean set union, $\cap$ to mean set intersection and $A-B$ to denote the set difference between A and B.

Definition 1. A *spider foot* is an element of the set $(\mathbb{Z}^+ \cup \{\bullet\}) \times \mathcal{Z}$ and the set of all feet is denoted $\mathcal{F}$. A spider foot $(k,z) \in \mathcal{F}$ has *rank* k where $k \in \mathbb{Z}^+ \cup \{\bullet\}$. The rank of a spider foot induces a relation $<$ on the feet, defined by $(k_1,z_1) < (k_2,z_2)$ if both $k_1,k_2 \in \mathbb{Z}^+$ and $k_1 < k_2$ hold or $k_1 = \bullet$ or $k_2 = \bullet$.

Whilst it may seems strange that $<$ as just defined is not a strict ordering (because $\bullet$ is both less than and greater than all other feet) this choice of $<$ simplifies many definitions.

Definition 2. A *spider*, s, is a non-empty set of feet together with a positive natural number, that is $s \in \mathbb{Z}^+ \times (\mathbb{P}\mathcal{F} - \{\varnothing\})$, and the set of all spiders is denoted $\mathcal{S}$. The set p is the *foot set* of spider $s=(n,p)$. The *habitat* of a spider $s=(n,p)$ is the region $\eta(s) = \{z : (k,z) \in p\}$.

In the semantics predicate, we will define a spider to represent an element corresponding to only one of its feet in one of the constituent regions of its habitat.

Formally, the set of all contour labels is denoted $\mathcal{C}$.

Definition 3. A *unitary spider diagram of order*, d, is a quadruple $\langle C(d), Z(d), ShZ(d), SI(d)\rangle$ where:

$C(d) \subseteq \mathcal{C}$ is a finite set of contour labels,
$Z(d) \subseteq \{(in, C(d)-in) : in \subseteq C(d)\}$ is a set of zones,
$ShZ(d) \subseteq Z(d)$ is a set of shaded zones,
$SI(d) \subsetneq \mathcal{S}$ is a finite set, called the *spider identifiers*, such that for all spiders $(n_1,p_1),(n_2,p_2)$ in $SI(d)$ if $p_1 = p_2$ then $n_1 = n_2$.

The set of spider identifiers, in effect, counts the number of spiders with a particular habitat. The set of *spiders* in d is defined to be

$$S(d) = \{(i,p) : (n,p) \in SI(d) \wedge 1 \le i \le n\}.$$

The symbol $\perp$ is also a unitary spider diagram. We define $C(\perp) = Z(\perp) = ShZ(\perp) = SI(\perp) = \varnothing$.

Spider diagrams of order may also be combined using the Boolean operations $\wedge$, $\vee$ and $\neg$. In addition we allow the binary connective $\triangleleft$. A spider diagram of order that contains one of the $\wedge$, $\vee$, $\neg$ or $\triangleleft$ connectives is a *compound* diagram. Furthermore, a spider diagram of order containing either no spiders or containing spiders consisting of only single feet is an α*–diagram*. A zone can be considered to be *missing* from a spider diagram as presented in [10].

Definition 4. Given an Euler diagram, d, a zone (in, out) is said to be *missing* if it is in the set $\{(in, C(d)-in) : in \subseteq C(d)\} - Z(d)$ with the set of such zones denoted $MZ(d)$. If d has no missing zones then d is in *Venn form*.

Spider diagrams of order have a model based semantics.

Definition 5. An *interpretation* is a triple $(U, \prec, \Psi)$ where U is a universal set and $\Psi : \mathcal{C} \to \mathbb{P}U$ is a function that assigns a subset of U to each contour label and $\prec$ is a strict total order on U. The function Ψ can be extended to interpret zones and sets of regions as follows:

1. each zone, $(in, out) \in \mathcal{Z}$, represents the set
$$\Psi(z) = \bigcap_{c \in in} \Psi(c) \cap \bigcap_{c \in out} (U - \Psi(c)) \quad \text{and}$$
2. each region, $r \in \mathbb{P}\mathcal{Z}$, represents the set which is the union of the sets represented by r's constituent zones, that is
$$\Psi(r) = \bigcup_{z \in r} \Psi(z).$$

Definition 6. Let $I = (U, \prec, \Psi)$ be an interpretation and let $d\ (\neq \perp)$ be a unitary spider diagram. Then I is a *model* for d, denoted $I \models d$, if and only if the following conditions hold.

1. *The missing zones condition*: All of the missing zones represent the empty set, that is, $\bigcup_{z \in MZ(d)} \Psi(z) = \varnothing$.
2. *The spider mapping condition*: There exists an injective function, $\varphi : S(d) \to U$ and a function $f : S(d) \to \mathcal{F}$, called a *valid pair*, such that the following conditions hold:
   (a) *The selected foot condition*: Each spider s must map, under f, to a spider foot in its foot set:
   $$\forall (n,p) \in S(d) f(n,p) \in p.$$
   (b) *The spiders' location condition*: All spiders represent elements in the sets represented by the zone in