Available online at www.sciencedirect.com
SCIENCE DIRECT®
Computer Vision and Image Understanding 99 (2005) 384–413
Computer Vision and Image Understanding
www.elsevier.com/locate/cviu

Tensor scale: A local morphometric parameter with applications to computer vision and image processing

Punam Kumar Saha *

Medical Image Processing Group, Department of Radiology, University of Pennsylvania, Philadelphia, PA 19104, USA

Received 4 September 2003; accepted 29 March 2005
Available online 24 May 2005

Abstract

Scale is a widely used notion in image analysis that evolved in the form of scale-space theory whose key idea is to represent and analyze an image at various resolutions. Recently, the notion of localized scale—a space-variant resolution scheme—has drawn significant research interest. Previously, we reported local morphometric scale using a spherical model. A major limitation of the spherical model is that it ignores structure orientation and anisotropy, and therefore fails to be optimal in many imaging applications including biomedical ones where structures are inherently anisotropic and have mixed orientations. Here, we introduce a new concept called "tensor scale"—a local morphometric parameter yielding a unified representation of structure size, orientation, and anisotropy. Also, a few applications of tensor scale in computer vision and image analysis, especially, in image filtering are illustrated. At any image point, its tensor scale is the parametric representation of the largest ellipse (in 2D) or ellipsoid (in 3D) centered at that point and contained in the same homogeneous region. An algorithmic framework to compute tensor scale at any image point is proposed and results of its application on several real images are presented. Also, performance of the tensor scale computation method under image rotation, varying pixel size, and background inhomogeneity is studied. Results of a quantitative analysis evaluating performance of the method on 2D brain phantom images at various levels of noise and blur, and a fixed background inhomogeneity are pre-

* Fax: +1 215 898 9145.
E-mail address: saha@mipg.upenn.edu.

1077-3142/$ - see front matter © 2005 Published by Elsevier Inc.
doi:10.1016/j.cviu.2005.03.003

sented. Agreement between tensor scale images computed on matching image slices from two 3D magnetic resonance data acquired simultaneously using different protocols are demonstrated. Finally, the application of tensor scale in anisotropic diffusive image filtering is presented that encourages smoothing inside a homogeneous region and also along edges and elongated structures while discourages blurring across them. Both qualitative and quantitative results of application of the new filtering method have been presented and compared with the results obtained by spherical scale-based and standard diffusive filtering methods.

Keywords: Scale-space; Local scale; Tensor; Edge location; Ellipse; Filtering; Anisotropic diffusion

1. Introduction

Computer vision and image understanding [1–10] is an active research area relating image enhancement, generation, perception, and their applications. *Scale* [2,11–13] may be thought of as the spatial resolution, or, more generally, a range of resolutions needed to ensure a sufficient yet compact object representation. Scale plays an important role in determining the optimum trade-off between noise smoothing and perception/detection of structures. Also, scale is helpful in breaking a computer vision and image-processing task into a hierarchy of tasks where tasks at higher levels deal with the larger structures. Marr [2] pointed out several benefits of *multi-scale* image analysis and computer vision. Witkin [11] and Koenderink [12] mathematically formulated the concept of scale in the form of *scale-space* theory. Discrete scale-space representations [13] have been used in several imaging applications including segmentation [14], clustering [15], classification [16], and structural analysis [17]. Although, scale-space image representations have provided significant insight, it is not obvious—(1) how to unify the information from images at different scales, and (2) how to identify the optimal scale at each individual image point. These questions have paramount implications on several computer vision and image analysis tasks. For example, a knowledge of "local scale" would allow us to spatially tune the neighborhood size in different processes leading to selection of small neighborhoods in regions containing fine detail or near a boundary, versus large neighborhoods in deep interiors [18]. Also, local scale would be useful in developing space-variant parameter controlling strategies [19] to improve the quality of results. These observations have stimulated interest in research relating to the concept of local scale [20–23]. Using the force field model, Tab and Ahuja [20] defined scale at each location as the minimum scale of the Gaussian weighting function at which the resultant attractive force exceeds a threshold (i.e., the force stabilizes). Pizer et al. [21] selected optimal local scale from the scale-space representation that maximizes "medialness" among the values at various scales. In their medialness model, local scale represents the distance at which a pair of opposing boundaries is expected. Elder and Zucker [22] described local scale as the smallest scale yielding a gradient measure above a statistically reliable threshold. Liang and Wang [23] proposed local scale as a measure of the number of zero crossings of the