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Extending anisotropic operators to recover smooth shapes ☆

Debora Gil*, Petia Radeva

Computer Vision Center (CVC), Edifici O, Campus UAB 08193 Bellaterra, Barcelona, Spain

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Abstract

Anisotropic differential operators are widely used in image enhancement processes. Recently, their property of smoothly extending functions to the whole image domain has begun to be exploited. Strong ellipticity of differential operators is a requirement that ensures existence of a unique solution. This condition is too restrictive for operators designed to extend image level sets: their own functionality implies that they should restrict to some vector field. The diffusion tensor that defines the diffusion operator links anisotropic processes with Riemmanian manifolds. In this context, degeneracy implies restricting diffusion to the varieties generated by the vector fields of positive eigenvalues, provided that an integrability condition is satisfied. We will use that any smooth vector field fulfills this integrability requirement to design line connection algorithms for contour completion. As application we present a segmenting strategy that assures convergent snakes whatever the geometry of the object to be modelled is.

Keywords: Contour completion; Functional extension; Differential operators; Riemmanian manifolds; Snake segmentation

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* Corresponding author.
E-mail addresses: debora@cvc.uab.es (D. Gil), petia@cvc.uab.es (P. Radeva).

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1. Introduction

The fact that solutions to a diffusion equation are infinitely differentiable has been largely used in image enhancement and restoration. However, the capability of smoothly extending functions to the whole image domain has been hardly exploited. Only in recent years some authors have used this property to extend vector fields [22] and, to some extend, to fill-in gaps in the image [2]. Object disocclusion, image restoration (in-painting), shape reconstruction, splines interpolation or, even, segmentation by snakes are some of the different fields in image processing that address the issue of line connection.

Object disocclusion and image in-painting focus on filling in gaps in an image in such a way that level lines arriving at gaps boundaries are smoothly prolonged inside. Following the Gestalt principle of good continuation, recent techniques ([2,5,17,18,12]) base on the way humans join unconnected curves to reconstruct the underlying shape. According to psychophysicist studies [14], the curves that reconstruct objects should minimize an energy functional involving length and curvature. To such purpose they seek for the function, which corresponds to the reconstructed image, that achieves a simultaneous minimum for all its level sets. The continuous variational problem includes a fourth order term [5] that troubles the numeric implementation. Solutions, up to our knowledge, include simplification of the functional, specific numeric algorithms [5], approximations to the solution [17] or the introduction [2] of an extra function to be minimized.

The second group of techniques (shape reconstruction and segmentation) deal with single contour completion. Shape recovery aims at obtaining a smooth model of shapes from a unconnected set of points, meanwhile object image segmentation must face connecting a set of points that lie on the contour of the objects of interest. An efficient way of modelling uncompleted shapes is by means of a snake [3,4,15]. Snakes or active contours correspond to minimums of an energy functional. Classic snakes [15] and geodesic snakes [3,4] are the two most popular energy designs recognized by the image processing community. Classic snakes produce smooth models of shapes but their convergence strongly depends on the definition of the external energy (commonly, distance or edge based map) and the snake parameterization. In spite of the improvements that the gradient vector flow (GVF)/generalized gradient vector flow (GGVF) [22] provides, classic snake convergence to concave curves is still poor. In the case of geodesic snakes, the internal and external energies join to produce a functional that measures the length of the snake in a Riemmanian manifold with the image gradient external energy as metric. Their level sets formulation [20] ensures convergence to an arbitrary number of objects. However, minimizing curve length leads to piece wise linear interpolation of the set of points and shock formation during the level sets evolution makes this formulation computationally expensive [19]. Moreover, balloon forces used to increase convergence to contours might push the snake into the object contour in the presence of large gaps.

In this paper, we address level sets completion for segmenting purposes. We consider the problem of image segmentation as the recovery of smooth models from a set of points, usually unconnected, on the objects of interest. The problem can be split in