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SCIENCE DIRECT
Image and Vision Computing 23 (2005) 799–806
www.elsevier.com/locate/imavis

Empirical mode decomposition synthesis of fractional processes in 1D- and 2D-space

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Received 9 February 2004; received in revised form 5 May 2005; accepted 5 May 2005

Abstract

We report here on image texture analysis and on numerical simulation of fractional Brownian textures based on the newly emerged Empirical Mode Decomposition (EMD). EMD introduced by N.E. Huang et al. is a promising tool to non-stationary signal representation as a sum of zero-mean AM-FM components called Intrinsic Mode Functions (IMF). Recent works published by P. Flandrin et al. relate that, in the case of fractional Gaussian noise (fGn), EMD acts essentially as a dyadic filter bank that can be compared to wavelet decompositions. Moreover, in the context of fGn identification, P. Flandrin et al. show that variance progression across IMFs is related to Hurst exponent H through a scaling law. Starting with these recent results, we propose a new algorithm to generate fGn, and fractional Brownian motion (fBm) of Hurst exponent H from IMFs obtained from EMD of a White noise, i.e. ordinary Gaussian noise (fGn with $H=1/2$).

Keywords: Empirical mode decomposition; Fractional processes synthesis; Gaussian and Brownian texture images

1. Introduction

This paper addresses the use of Bidimensional Empirical Mode Decomposition (BEMD), a 2D extension [1,2] of the so-called EMD proposed by N.E. Huang et al. [3], to simulate fractional processes. Many natural processes such as weather data, optical, electrical or physiological measurements, and man made phenomena such as traffic flow data exhibit spectrums which have been observed to follow the $1/f$ law. For this reason, fractional Brownian motion (fBm) processes are very important because the spectrum follows a $1/f$ power law. In fact, fBm is the only known correlation model that satisfies Wornell's definition of $1/f$ processes [4]. One characterizing feature of fBm is its statistical self-similar property [5], which means that the variance of increments of a process obeys a hyperbolic scaling law so that the statistical properties of the process at any two scales are the same within a scaling constant. The exponent of the hyperbolic law deals with the Hurst parameter H that quantifies the persistence of an fBm realization. In fractal theory, Hurst exponent is related to the fractal dimension in a simple manner [6]. The fBm model has been successfully used in texture analysis and synthesis, landscape modeling and speech segmentation. Especially, in computer vision, Brownian texture is a widely used Gaussian process with a variety of applications in image analysis, e.g. in physics, medical and fractal imaging.

The estimation of the fractal dimension, or of the Hurst exponent of an fBm realization has proven to be an important problem both for signal and image analysis. Traditional techniques are based on some type of regression analysis to measure the hyperbolic progression of the average size of increments at varying scales, the progression of power versus frequency, or the progression of the variance of the wavelet coefficients at different scales [7–9]. A maximum likelihood fractal and fGn estimators were proposed in [10,11]. More recently, an estimator for noisy fBm measurements that takes advantage of the decorrelation effects of orthogonal wavelets was published in [12,13].

There exist algorithms for simulating general fractional processes with a given autocovariance function. These algorithms can be used for generating 1D signals or texture images. Textures and natural data are often modeled by the increments of the sampled fBm, known as (discrete)

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doi:10.1016/j.imavis.2005.05.012

fractional Gaussian noise (fGn). Therefore, fBm and fGn, or simply fractal models, have been successfully applied to texture analysis and synthesis [9,14–16] and terrain modeling [17–19]. For a general point of view, there exist several methods for generating long-range dependent processes, such as fBm or fGn. Two classes of methods can be defined; (i) Cholesky Decomposition methods based on fGn covariance matrix, and (ii) spectral [20], wavelets [7] or autoregressive models [21] methods. See [22] for a survey about these methods.

Both comparison of proposed synthesis method with others published methods and fully validation are beyond the scope of this paper. We just reported here that EMD method might offer a new way to synthesis fractional processes in 1D and 2D space.

2. Empirical mode decomposition and fractional processes

2.1. EMD method

Applying the EMD method will generate a collection of Intrinsic Mode Functions (IMFs). The decomposition is based on direct extraction of the energy associated with various intrinsic time scales. The combination of the EMD method and associated Hilbert spectral analysis [3] can offer a powerful method for non-linear and non-stationary data analysis. We summarized here the Empirical Mode Decomposition method. Details of the implementation of EMD algorithm and Matlab codes are fully available in [1,3,23,24]. The central idea of the EMD is the sifting process to decompose any given signal into its fundamental modes. With this approach, the basis functions themselves are non-linear functions, which can be extracted directly from the data. So, an adaptive basis called Intrinsic Mode Function (IMF) can be found. To be an IMF, a signal must satisfy two criteria, the first one being that the number of local maxima and the number of local minima differ by at most one, and the second, the mean of its upper and lower envelopes equals zero.

For any signal, $s(t)$, EMD ends up with the following representation

$$s(t) = r^{\langle k\rangle}(t) + \sum_{k=1}^{K} C^{\langle k\rangle}(t), \qquad (1)$$

where $C^{\langle k\rangle}(t)$ is the kth mode (or IMF) of the signal, and $r^{\langle K\rangle}(t)$ stands for residual trend (a low order polynomial component).

Sifting procedure produces a finite (and limited) number of modes that are zero-mean AM-FM components. Functions $C^{\langle k\rangle}(t)$ are nearly orthogonal to each other, and by nature of the decomposition procedure, the technique decomposes data into K fundamental component, each with distinct time scale: the first component has the smallest time

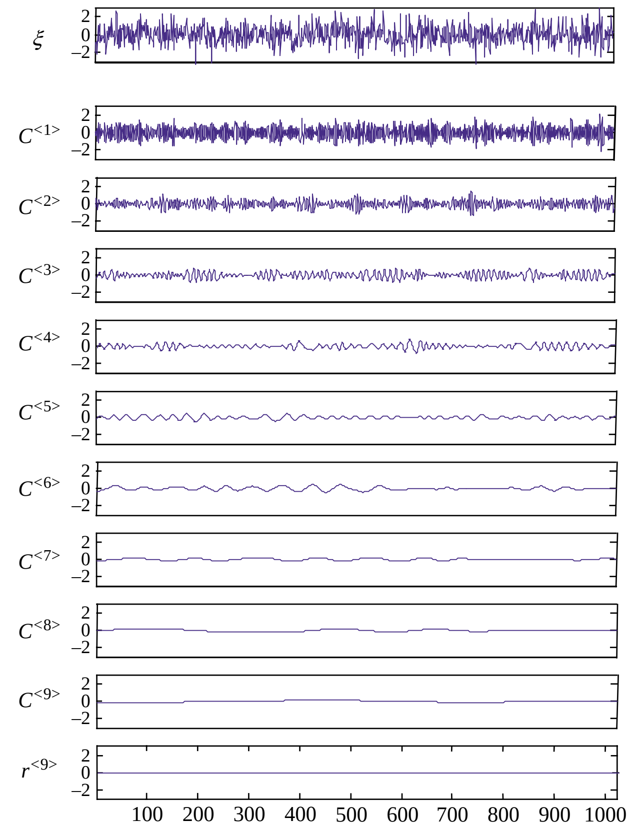

Fig. 1. Empirical Mode Decomposition of a White noise realization ($N = 2^{10}$). Upper trace, White noise $\xi \in N(0,1)$. Lower traces, IMFs $C^{\langle k\rangle}$ ($k = 1,\ldots,9$) and residue $r^{\langle 9\rangle}$.

scale, which correspond to the fastest time variation of the data. As the decomposition proceeds, the time scale increases, and hence, the mean frequency of the mode decreases.

An example of decomposition is illustrated in Fig. 1. The original signal is a White noise realization of data length

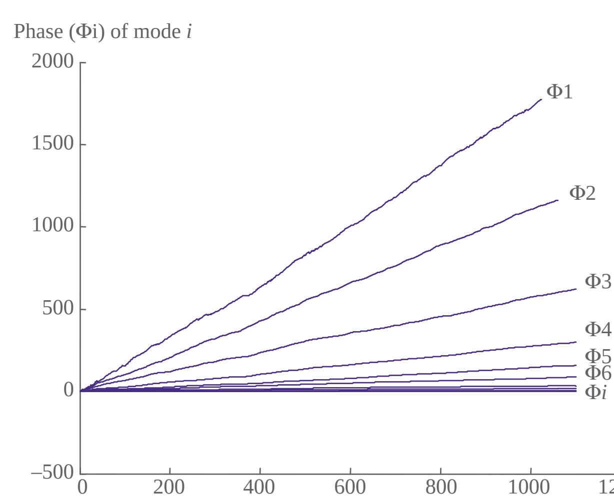

Fig. 2. Examples of EMD-based simulation of fractional processes. Top, fGn generated processes for different Hurst exponent values, $H = 0.2$, 0.5 and 0.8 from left to right. Bottom, fBm generated processes.