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A smoothness constraint set based on local statistics of BDCT coefficients for image postprocessing

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Abstract

In blocking artifacts reduction based on the projection onto convex sets (POCS) technique, good constraint sets are very important. Until recently, smoothness constraint sets (SCS) are often formulated in the image domain, whereas quantization constraint set is defined in the block-based discrete cosine transform (BDCT) domain. Thus, frequent BDCT transform is inevitable in alternative projections. In this paper, based on signal and quantization noise statistics, we proposed a novel smoothness constraint set in the BDCT transform domain via the Wiener filtering concept. Experiments show that POCS using this smoothness constraint set not only has good convergence but also has better objective and subjective performance. Moreover, this set can be used as extra constraint set to improve most existing POCS-based image postprocessing methods.

Keywords: Projection onto convex sets; Postprocessing methods; BDCT

1. Introduction

The block-based discrete cosine transform (BDCT) [1] has been used widely in image and video compression. To reduce the bit-rate, the coefficients of BDCT are often quantized. At low bit rate, this causes annoying blocking artifacts in the decoded image. Recently, several post-processing methods have been proposed to alleviate blocking artifacts. Postprocessing techniques are attractive because they are independent of coding schemes and can be applied to commonly used JPEG [1], H.263, and MPEG compression standards.

The approach based on the theory of POCS has a major advantage in that it can exploit the a priori knowledge about the image. If the convex constraints sets associated with the image information can be found, the POCS algorithm with corresponding projectors will converge to the intersection of all the constraint sets. In the past, various constraint sets have been proposed. Generally, these constraint sets can be classified into two categories. One is the quantization constraint set (QCS) [2,3], and the other is the smoothness constraint set (SCS) [4,5]. However, most SCS are implemented in the image domain, whereas QCS are defined in the BDCT domain. Therefore, a BDCT transform of the whole image is needed in each iteration. This incurs high computational cost. Although there are some filtering methods available that work in BDCT domain [6], they are not POCS-based and it is difficult to incorporate new a priori knowledge.

In this paper, we proposed a new SCS, which is defined in the BDCT domain. The new SCS is derived from signal and quantization noise statistics and uses a least mean square formulation based on the Wiener filter. Experiments show that POCS using this SCS not only has faster convergence but also has better objective and subjective performance. Moreover, this new SCS can be used as a new constraint set to improve most of the available POCS-based image postprocessing algorithms.

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2. Mathematical background

2.1. POC-based image reconstruction

In POCS-based image post-processing [7], every known a priori property about the original image can be formulated as a corresponding convex set in a Hilbert space H. Given n closed convex set C_i, $i=1,2,\ldots,n$, and $C_0=\cap_{i=1}^{m}C_i$ nonempty, the iteration

$$\mathbf{x}_{k+1} = P_m P_{m-1} \ldots P_1 \mathbf{x}_k, \quad k = 0, 1, 2, \ldots \tag{1}$$

where P_i is the projector onto C_i defined by

$$||\mathbf{x} - P_i\mathbf{x}|| = \min_{g \in C_i} ||\mathbf{x} - \mathbf{g}|| \tag{2}$$

and g is the projection of x onto C_i, will converge to a point in C_0 for any initial x_0.

2.2. The mathematical model of image deblocking problem

Throughout this paper we use the following conventions: a real $N\times N$ image $\mathbf{x}$ can be treated as an $N^2\times 1$ vector in the space R^{N^2} by lexicographic ordering by either rows or columns. Then, the 2D DCT transform can be expressed as:

$$X = T\mathbf{x}, \quad \text{and} \quad \mathbf{x} = T^{-1}X \tag{3}$$

where X is the BDCT coefficients of x and T is the BDCT transform matrix.

In order to lower the bit-rate, X is quantized. Let Q denote the quantization process. The BDCT coefficients suffering lossy quantization can be denoted by

$$Y = QT\mathbf{x} \tag{4}$$

The decoded image with blocking artifacts is given by $\mathbf{y}=T^{-1}QT\mathbf{x}$. If a uniform scalar quantizer is used, then Y can be expressed as

$$Y = X + \mathbf{n} \tag{5}$$

where $\mathbf{n}$ is the additive zero mean noise introduced by the quantizer and contributes to the blocking artifacts in the encoded image. The POCS deblocking problem thus involves the estimation of X from Y using the available information about the quantizer and the image, formulated as convex constraint sets.

3. Proposed postprocessing technique

It is well known that the least mean square error solution for Eq. (5) is Wiener filtering. Specifically, the locally adaptive Wiener filter [8], which is capable of tracking the signal and noise characteristics over different image regions, can be used to estimate the true BDCT coefficients by

$$\hat{X}_i = \bar{X}_i + \frac{\sigma_{X_i}^2}{\sigma_{X_i}^2 + \sigma_{n_i}^2}(Y_i - \bar{X}_i) \tag{6}$$

where $\bar{X}$ is the a prior mean of X, X, $\bar{X}$and Y are treated as an $N^2\times 1$ vector in the space R^{N^2} by lexicographic ordering by either rows or columns of their 2D versions. By defining two matrix M and R,

$$R = \begin{bmatrix} \frac{1}{\sigma_{n_1}^2} & 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{\sigma_{n_2}^2} & 0 & \cdots & \vdots \\ 0 & 0 & \ddots & 0 & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & \frac{1}{\sigma_{n_{N\cdot N}}^2} \end{bmatrix} \qquad M$$

$$= \begin{bmatrix} \frac{1}{\sigma_{X_1}^2} & 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{\sigma_{X_2}^2} & 0 & \cdots & \vdots \\ 0 & 0 & \ddots & 0 & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & \frac{1}{\sigma_{X_{N\cdot N}}^2} \end{bmatrix} \tag{7}$$

Eq. (6) can be written in the matrix form

$$\hat{X} = \bar{X} + R(M + R)^{-1}(Y - \bar{X}) \tag{8}$$

Although adaptive Wiener filtering is effective in image deblocking, it has an apparent shortcoming. If we transform and quantize the output image, what we obtain does not equal to the original quantized coefficients. This violates our information about the original image apparently. In [9], a method combining low-pass filtering and QCS is provided to limit the output of low-pass filter to conform to the quantized information. However, in [10], this method is approved to be non-convergent unless an ideal low pass filter is used. As is well known, ideal low pass filter is impossible to realize.

To solve this problem, we proposed a new SCS to replace the filtering. It is derived from least mean square formulation based on the Wiener filter and thus retains its effectiveness in image deblocking.

In fact, Eq. (8) is the solution to the following regularization problem [6]:

$$J = (X - \bar{X})^t M(X - \bar{X}) + (Y - X)^t R(Y - X) \tag{9}$$

The first term in Eq. (9) accounts for image smoothness, whereas the second term ensures image fidelity. According to the POCS theory, we do not need to obtain the solution minimizing Eq. (9), which clearly depends on our estimate of $\bar{X}$, M and R. Instead, we only need to find a set that includes the original image. A reasonable choice is to limit J with a threshhold value E_a, that is $|J|\le E_a$, where E_a is a value larger than but close enough to the minimum of J so that all the images satisfying the condition are smooth and