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Fuzzy spatial relationships for image processing and interpretation: a review

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Abstract

In spatial reasoning, relationships between spatial entities play a major role. In image interpretation, computer vision and structural recognition, the management of imperfect information and of imprecision constitutes a key point. This calls for the framework of fuzzy sets, which exhibits nice features to represent spatial imprecision at different levels, imprecision in knowledge and knowledge representation, and which provides powerful tools for fusion, decision-making and reasoning. In this paper, we review the main fuzzy approaches for defining spatial relationships including topological (set relationships, adjacency) and metrical relations (distances, directional relative position).

Keywords: Fuzzy spatial relationships; Degree of intersection; Degree of inclusion; Degree of adjacency; Distances; Directional relative position; Structural pattern recognition; Image interpretation; Spatial reasoning

1. Introduction

Spatial reasoning can be defined as the domain of spatial knowledge representation, in particular spatial relations between spatial entities, and of reasoning on these entities and relations. This field has been largely developed in artificial intelligence, in particular using qualitative representations based on logical formalisms. In image interpretation and computer vision, it is much less developed and is mainly based on quantitative representations. A typical example in this domain concerns model-based structure recognition in images. The model constitutes a description of the scene where objects have to be recognized. This description can be of iconic type, as for instance a digital map or a digital anatomical atlas, or of symbolic type, as linguistic descriptions of the main structures. The model can be attached to a specific scene, the typical example being a digital map used for recognizing structures in an aerial or satellite image of a specific region. It can also be more generic, as an anatomical atlas, which is a schematic representation that can be used for recognizing structures in a medical image of any person. In both types of descriptions (iconic and symbolic), objects are usually described through some characteristics like shape, size, appearance in the images, etc. But this is generally not enough to discriminate all objects in the scene, in particular if they are embedded in a complex environment. For instance in a magnetic resonance image of the brain, several internal structures appear as smooth shapes with similar grey levels, making their individual recognition difficult. Similar examples can be found in other application domains. In such cases, spatial relationships play a crucial role, and it is important to include them in the model in order to guide the recognition [1]. The importance of spatial relationships has been similarly recognized in many different works. Many authors have stressed the importance of topological relationships, but distances and directional relative position are also important. Freeman [2] distinguishes the following primitive relationships: left of, right of, above, below, behind, in front of, near, far, inside, outside, surround. Kuipers [3,4] considers topological relations (set relations, but also adjacency which was not considered by Freeman)

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and metrical relations (distances and directional relative position). In this paper, we will consider all these relations.

Moreover, imprecision has to be taken into account in such problems. Imprecision is often inherent to images, and its causes can be found at several levels: observed phenomenon (imprecise limits between structures or objects), acquisition process (limited resolution, numerical reconstruction methods), image processing steps (imprecision induced by a filtering procedure for instance). This may induce imprecision on the objects to be recognized (due to the absence of strong contours or to a rough segmentation). But imprecision can be found also in semantics of some relationships (such as 'left of', 'quite far', etc.), or in the type of knowledge available about the structures (for instance anatomical textbooks describe the caudate nucleus as 'an internal brain structure which is very close to the lateral ventricles') or even in the type of question we would like to answer (in mobile robotics for instance, we may want a robot 'go towards an object while remaining at some security distance of it').

In summary, the main ingredients in problems related to spatial reasoning include knowledge representation (including spatial relationships), imprecision representation and management, fusion of heterogeneous information and decision-making. Fuzzy set theory is of great interest to provide a consistent mathematical framework for all these aspects. It allows to represent imprecision of objects, relationships, knowledge and aims, it provides a flexible framework for information fusion as well as powerful tools for reasoning and decision-making.

The aim of this paper is to review the main approaches for modeling spatial relationships under imprecision in the fuzzy set framework. We distinguish between relationships that are mathematically well defined and relationships that are intrinsically vague. Topological relationships (such as set relationships and adjacency) and distances belong to the first class. If the objects are precisely defined, their relationships can be defined and computed in a numerical (purely quantitative) setting. But if the objects are imprecise, as is often the case if they are extracted from images, then the semi-quantitative framework of fuzzy sets proved to be useful for their representation, as spatial fuzzy sets. Definitions of relationships have then to be extended to be applicable on fuzzy objects. Results can also be semi-quantitative, and provided in the form of intervals or fuzzy numbers. Some metric relationships, like relative directional position, belong to the second class. Even for crisp objects, fuzzy definitions are then appropriate.

Section 2 contains some preliminaries about spatial fuzzy sets, some basic definitions, and general principles to extend a crisp relation to a fuzzy one. Set theoretical relationships (intersection and inclusion) are described in Section 3. Then, other topological relations (local such as neighborhood or more global such as adjacency) are addressed in Section 4. Distances are reviewed in Section 5 and finally directional relative position in Section 6.

2. Preliminaries

2.1. Spatial fuzzy sets

Let $\mathcal{S}$ be the image space, typically $\mathbb{Z}^2$ or $\mathbb{Z}^3$ for digital 2D or 3D images, or, in the continuous case, $\mathbb{R}^2$ or $\mathbb{R}^3$. A spatial fuzzy set (or fuzzy image object) is a fuzzy set defined on $\mathcal{S}$. Its membership function μ represents the imprecision in the spatial extent of the object. For any point x of $\mathcal{S}$ (pixel or voxel), $\mu(x)$ is the degree to which x belongs to the fuzzy object. As usual in the fuzzy set community, and for the sake of simplicity, μ will denote both the fuzzy set and its membership function. Using fuzzy sets may represent different types of imprecision, either on the boundary of the objects (due for instance to some partial volume effect or to the spatial resolution), or on their individual variability, etc. In the sequel, $\mathcal{F}$ denotes the set of all fuzzy sets defined on $\mathcal{S}$.

2.2. Notations and basic definitions

We recall here a few basic definitions for the sake of completeness and introduce some notations. A complete description of fuzzy set theory can be found in Ref. [5]. From $\mu \in \mathcal{F}$, some particular crisp (binary) sets can be derived, such as its core: $\mathrm{Core}(\mu) = \{x \in \mathcal{S}, \mu(x) = 1\}$, its support: $\mathrm{Supp}(\mu) = \{x \in \mathcal{S}, \mu(x) > 0\}$, its α-cuts (for $\alpha \in]0,1]$): $\mu_\alpha(x) = \{x \in \mathcal{S}, \mu(x) \geq \alpha\}$.

A fuzzy number is a convex upper semi-continuous (and unimodal) fuzzy set on $\mathbb{R}^+$ having a bounded support.

A few basic operators on membership values will be used in the following, such as t-norrns, t-conorms and complementation [5]. A t-norm is an operator t from $[0,1] \times [0,1]$ into $[0,1]$ which is commutative, associative, increasing in both variables and that admits 1 as unit element. It represents a conjunction and generalizes intersection and logical 'and'. Typical examples are $\min(a, b)$, ab, $\max(a+b-1, 0)$, the last one being known as the Lukasiewicz t-norm. A t-conorm is an operator T from $[0,1] \times [0,1]$ into $[0,1]$ which is commutative, associative, increasing in both variables and that admits 0 as unit element. It represents a disjunction and generalizes union and logical 'or'. Typical examples are $\max(a, b)$, $a+b-ab$, $\min(a+b, 1)$, the last one being the Lukasiewicz t-conorm. A complementation is an operator c from $[0,1]$ into $[0,1]$ which is strictly decreasing, involutive, and such that $c(0)=1$, $c(1)=0$. The most used complementation is defined as $\forall a \in [0,1]$, $c(a) = 1-a$. From a t-norm t and a complementation c a dual t-conorm T can be derived as $\forall (a,b) \in [0,1] \times [0,1]$, $T(a, b) = c(t(c(a), c(b)))$. A strictly monotonous Archimedian t-norm is a t-norm t such that $\forall a \in [0,1]$, $t(a,a) < a$ and $\forall (a,b,b') \in [0,1]^3$, $b < b' \Rightarrow t(a,b) < t(a,b')$. Archimedian t-conorms are defined by duality. A typical example of Archimedian t-norm is the product.

We will denote by R_B a relation between two binary (crisp) subsets of $\mathcal{S}$. This relation can provide a binary