Pattern Recognition 47 (2014) 1144–1158

Contents lists available at ScienceDirect

Pattern Recognition

journal homepage: www.elsevier.com/locate/pr

Robust similarity registration technique for volumetric shapes represented by characteristic functions

Wanmu Liu [a], Sasan Mahmoodi [a,*], Tom Havelock [b,c], Michael J. Bennett [c]

[a] School of Electronics and Computer Science, University of Southampton, Southampton SO17 1BJ, United Kingdom
[b] Faculty of Medicine, University of Southampton, Southampton SO16 6YD, United Kingdom
[c] Southampton NIHR Respiratory Biomedical Research Unit, Southampton University Hospital NHS Foundation Trust, Southampton SO16 6YD, United Kingdom

ARTICLE INFO

Article history:
Received 23 January 2013
Received in revised form
10 August 2013
Accepted 13 August 2013
Available online 8 October 2013

Keywords:
Similarity registration
Volumetric shapes
Characteristic functions
Spherical cross-correlation
Phase correlation
Registration of lungs

ABSTRACT

This paper proposes a novel similarity registration technique for volumetric shapes implicitly represented by characteristic functions (CFs). Here, the calculation of rotation parameters is considered as a spherical cross-correlation problem and the solution is therefore found using the standard phase correlation technique facilitated by principal components analysis (PCA). Thus, fast Fourier transform (FFT) is employed to vastly improve efficiency and robustness. Geometric moments are then used for shape scale estimation which is independent from rotation and translation parameters. It is numerically demonstrated that our registration method is able to handle shapes with various topologies and robust to noise and initial poses. Further validation of our method is performed by registering a lung database.

1. Introduction

1.1. Background

Shape registration is a fundamental technique in computer vision. It benefits a variety of research fields, such as statistical shape study, shape-based image segmentation, 3-D modeling of real world objects, which give rise to all kinds of registration techniques. Therefore, before a certain registration technique is developed, several aspects need to be taken into account:

Shape representations: The raw data acquired at hand may well differ in various research fields, and they are intended to be well suited for application purposes. For example, in real world object modeling, popular representations include range data [13,8,14] and point sets [5,3], while in shape-based image segmentation [25,26], parametric surfaces and signed distance functions (SDFs, [16]) are frequently used for curve/surface evolutions. In medical imaging, shapes are often represented by characteristic functions (CFs) which serve as mask to emphasize regions of interest.

Expected results of registration: In 3-D object modeling, a sequence of partial views represented by range data are to be registered for acquisition of a full 3-D object. This process involves matching of common regions of surfaces. In shape-based image segmentation and statistical shape modeling, registration finds a suitable match of two entire shapes.

Degrees of precision: In fact, registration precision is closely associated with given degrees of freedom. Rigid transformation [3,8,2] involves rotation and translation and similarity registration further includes scale, while non-rigid transformation allows local deformation to achieve a greater matching. One can choose proper degrees of freedom according to the registration problem at hand. For example, entire shape or surface may be sufficiently matched by rigid transformation, however, more sophisticated modeling such as facial expression modeling or heart ventricle motion tracking involves non-rigid transformation [11].

Similarity/dissimilarity measure of registration: Most frequently used measure is a sum of squared distance/difference (SSD) between either explicit corresponding points or functions that used to represent shapes without explicit correspondence. Novel measures used in recent years include information theoretic measure between probability distribution functions estimated from point sets or signed distance functions [5,11,29] and kernel correlation of point sets entropy [28]. There are all kinds of measures to choose from, however, the measure should be well suited for the representations of shapes to achieve a satisfying result for registration.

The method proposed here is intended to be applied to two areas of research: statistical shape study of volumetric shapes and shape-based volumetric image segmentation. Considering the four aspects given above, the method could be described as follows.

* Corresponding author. Tel: +44 23 8059 3284; fax: +44 23 8059 4498.
E-mail addresses: wl3g10@ecs.soton.ac.uk (W. Liu), sm3@ecs.soton.ac.uk (S. Mahmoodi), t.havelock@soton.ac.uk (T. Havelock), michael.bennett@soton.ac.uk (M.J. Bennett).

0031-3203/$ - see front matter
http://dx.doi.org/10.1016/j.patcog.2013.08.013

It concerns shapes represented by CFs. Anatomical structures acquired from medical images are mostly volumetric and characteristic functions that denote the regions of interest in a straightforward way. Also, in shape-based image segmentation, regions inside the reference shapes and the target areas of images are to be maximized, where CFs are a representation with simplicity.

It registers two entire shapes. In statistical modeling of volumetric shapes, one entire shape in volumetric image could be obtained through manual or computer-aided segmentation, which does not involve matching partial views (range data) together. The same goes with shape-based image segmentation, the target areas are intact shapes and they are to be registered with model shapes at hand.

It handles similarity registration that involves isometric scale, rotation and translation. Similarity registration is a precursor to statistical modeling of volumetric shapes. Furthermore, in the shape-based image segmentation, the first step to bring reference shape to segmentation process is similarity registration before any local deformation.

It uses region-based similarity measures between shapes. Region-based similarity measures between CFs denote the volume of homogeneous region of shapes. In statistical modeling of volumetric shapes, we focus more on the region features rather than the boundaries. With regard to the volumetric image segmentation, although it is ideal to achieve both region and boundary accuracy, the final results one would expect first are accurate regions. It should be pointed out that the registration is performed using all voxels inside shapes to be registered.

Let us now proceed to review several previous works that concern this topic.

1.2. Previous works

The method frequently used similarity registration is gradient descent optimization of shapes represented by SDFs. The shape-based segmentation method proposed in [6] involves rigid registration in 2-D using a variational framework. It handles registration by optimizing a proposed functional that iteratively registers the evolving contours to rigid shapes represented by level-set functions. Similar works are done in [27,4], who applied rigid/affine registration to statistical modeling of shapes and image segmentation process. Also, in [17], similarity registration is used as a pre-alignment technique for non-rigid registration. These works commonly choose SSD as the measure for similarity between shapes, which also suffer from local minima problem. Alternatively, it is proposed in [11] to maximize mutual information between SDFs of shapes and the method performs well in finding a global maximum.

The methods proposed in [1,15] have a close relationship with the works done in this paper. The calculations of rotation and translation parameters are related to standard correlation problems and scale parameter is computed using geometric metric moments of shapes. Experiments show that they have good robustness against occlusions, noise and topological differences. However, these methods are based on 2D and shapes are represented by SDFs, while the method proposed in this paper employs CFs as shape representation and designed for volumetric shapes.

Another method that concerns this topic is the iterative closed point (ICP) method introduced in [2] that solves general rigid registration problem (concerning rotation and translation). This method, at each iteration, finds the closest points on the surface of target shape to that of the reference shape and optimizes rotation and translation. Results indicate that it is suitable for a variety of representations of shapes including point sets, parametric surfaces and implicit surfaces represented by level-set functions. The ICP method performs well in local optimization, however, when the poses of shapes to be registered have large differences, it may fall into local minima. Another disadvantage is that it is claimed to be slow. Accelerated ICP methods were later proposed in [21,9,23].

In recent years, the Laplace–Beltrami spectra employed as shape descriptors in [20,19,18,22] could be used to perform analyses of shapes regardless of their poses and scale. Our method has some similarities with these works. In the calculations of rotation and scale parameters, shapes are transformed into other representations. However, these works aim at evaluating similarity between shapes without registration for shape retrieval from database and quality assessment of data that represent surfaces and volumes.

The rest of the paper is organized as follows. Section 2 presents some mathematical preliminaries and statement of the problem concerning shape registration. Section 3 describes the theory behind the registration technique proposed here as well as some implementation remarks. Experimental results are presented in Section 4. Finally we conclude the paper and give future directions of the method in Section 5.

2. Mathematical preliminaries

2.1. Unit quaternions as representation of rotations

Unit quaternions are used as a mathematical representation of rotation of volumetric rigid shapes. A unit quaternion is a four vector $\vec{q} \in \mathbb{S}^3$, where $\mathbb{S}^3 = \{\mathbf{h} \in \mathbb{R}^4 : \|\mathbf{h}\| = 1\}$ ($\|\cdot\|$ is the Euclidean norm). $\mathbb{S}^3$ represents a unit sphere in 4-D Euclidean space, frequently referred to as unit 3-sphere. A volumetric rigid body could be considered as a set of 3-D vectors, and rotation of a vector set about a fixed axis is a linear transform and performed by a 3×3 matrix, denoted by $\mathbf{R}$ with $\det(\mathbf{R}) = 1$. It is explained well in [7] that one unit quaternion $\vec{q}$ generates one rotation matrix $\mathbf{R}$ through the equation given in the footnote.[1] Next, we give a brief review of quaternion representation of rotation.

A unit quaternion in representation of rotation consists of a scalar part and a vector part, namely $\vec{q} = (\eta, \vec{v})$. It has a counterpart known as the 'conjugate', denoted by $\vec{q}^* = (\eta, -\vec{v})$. Multiplication of quaternions follows the formula below:

$$\vec{q}_1 \vec{q}_2 = (\eta_1\eta_2 - \vec{v}_1 \cdot \vec{v}_2, \eta_1 \vec{v}_2 + \eta_2 \vec{v}_1 + \vec{v}_1 \times \vec{v}_2) \tag{1}$$

where '·' is the inner product of vectors and '×' their cross product. However, this multiplication is non-commutative. Multiplication of a set of unit quaternions in a particular order, namely $\vec{q} = \vec{q}_1 \vec{q}_2 \cdots \vec{q}_N$, could produce one unique rotation and the corresponding inverse rotation could be produced by $\vec{q}^* = \vec{q}^*_N \cdots \vec{q}^*_2 \vec{q}^*_1$ (see Fig. 1 for an example).

More intuitively, a unit quaternion consists of an axis $\vec{a} \in \mathbb{S}^2$, where $\mathbb{S}^2 = \{\hat{\mathbf{x}} \in \mathbb{R}^3 : \|\hat{\mathbf{x}}\| = 1\}$, and an angle $\Delta\theta \in \mathbb{R}$, making $\vec{q}(\vec{a}, \Delta\theta) = (\cos(\Delta\theta/2), \vec{a}^{\mathrm{T}} \sin(\Delta\theta/2))$. Vector $\vec{a}$ and angle $\Delta\theta$ are considered as the axis and angle of rotation, following the right handle rule.

[1] Assuming that $\vec{q} = (q_0, q_1, q_2, q_3)$,

$$\mathbf{R}(\vec{q}) = \begin{pmatrix} q_0^2+q_1^2-q_2^2-q_3^2 & 2(q_1q_2-q_0q_3) & 2(q_1q_3+q_0q_2) \\ 2(q_1q_2+q_0q_3) & q_0^2+q_2^2-q_1^2-q_3^2 & 2(q_2q_3-q_0q_1) \\ 2(q_1q_3-q_0q_2) & 2(q_2q_3+q_0q_1) & q_0^2+q_3^2-q_1^2-q_2^2 \end{pmatrix}$$