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Intrinsic dimension estimation via nearest constrained subspace classifier

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ABSTRACT

We consider the problems of classification and intrinsic dimension estimation on image data. A new subspace based classifier is proposed for supervised classification or intrinsic dimension estimation. The distribution of the data in each class is modeled by a union of a finite number of affine subspaces of the feature space. The affine subspaces have a common dimension, which is assumed to be much less than the dimension of the feature space. The subspaces are found using regression based on the ℓ_0-norm. The proposed method is a generalisation of classical NN (Nearest Neighbor), NFL (Nearest Feature Line) classifiers and has a close relationship to NS (Nearest Subspace) classifier. The proposed classifier with an accurately estimated dimension parameter generally outperforms its competitors in terms of classification accuracy. We also propose a fast version of the classifier using a neighborhood representation to reduce its computational complexity. Experiments on publicly available datasets corroborate these claims.

1. Introduction

The concept of data manifold plays a vital role in pattern recognition. Briefly speaking, a data manifold is a topological space which contains the data samples, and which serves as an ideal geometric description of the data. In this description, all data points, including the observed and unobserved, lie in a data manifold, whose dimension is often much lower than the dimension of the feature space which contains it.

In previous work, the manifold model has been used as a powerful analytical approximation tool for nonparametric signal classes such as human face images or handwritten digits [1–3]. If the data manifold is learned, then it can be exploited for classifier design. The manifold learning usually involves constructing a mapping from the feature space to a lower-dimensional space that is adapted to the training data and that preserves the proximity of data points to each other.

There have been many works on manifold learning. For example, methods such as ISOMAP (ISOmetric Mapping) [4], Hessian Eigenmaps (also known as HLLE, Hessian Locally Linear Embedding) [5], LLE (Local Linear Embedding) [6], Maximum Variance Unfolding (MVU) [7], Local Tangent Space Alignment (LTSA) [8] and Laplacian Eigenmap [9] have been introduced. These methods learn a low-dimensional manifold under the constraint that the proximity properties of the nearby data are preserved.

We propose a novel supervised classifier framework in which each class is modeled by a union of a finite number of affine subspaces. The proposed algorithm is superior to the traditional classifiers such as NN (Nearest Neighbor), NFL (Nearest Feature Line, proposed by Li [10]), NS (Nearest Subspace), etc., because the use of finite affine subspaces allows a more accurate description of the distribution of the data.

The reminder of this paper is organized as follows. In Section 2, some background and related works about the classical classifiers including NN (Nearest Neighbor), NFL (Nearest Feature Line) and NS (Nearest Subspace) are briefly revisited. In Section 3, the classification model of NM (Nearest Manifold) and some classifier design principles are presented. Then, a novel constrained subspace framework named NCSC and its fast version are proposed in Section 4. Section 5 gives the experimental results on several publicly available datasets. In Section 6, some concluding remarks are given.

2. Background and related works

We argue that the NN, NFL and NS classifiers can be incorporated into a unified framework. Before the detailed discussion, let us first briefly revisit the theoretical background.

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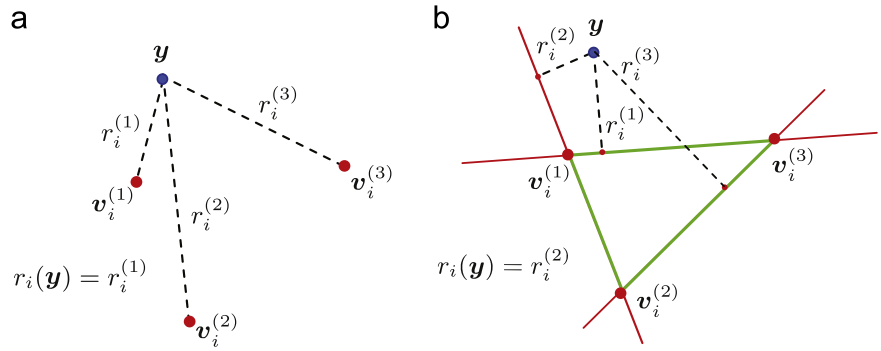

Fig. 1. The distances of a query sample $\mathbf{y}$ in NN and NFL to a class (the i-th, as shown), where $N_i=3$. (a) NN and (b) NFL.

2.1. NN and NFL

The NN, NFL and NS classifiers base the classification of a sample $\mathbf{y}$ on the distances measured in the feature space.

For NN and NFL, there exists a convenient geometrical interpretation — given N_i training samples in a given class (say, the i-th class), the distances are obtained as illustrated in Fig. 1 where to simplify the explanations, we set $N_i=3$.

In Fig. 1a, the distance from $\mathbf{y}$ to the i-th class is the minimum $r_i(\mathbf{y})$ of the distances from $\mathbf{y}$ to the training samples in the i-th class. In Fig. 1b, each pair of training samples defines a line. The distance from $\mathbf{y}$ to the i-th class is defined as the minimum $r_i(\mathbf{y})$ of the distances from $\mathbf{y}$ to the different lines.

More generally, given the training samples $\mathbf{v}_i^{(1)}, \cdots, \mathbf{v}_i^{(N_i)}$ of the i-th class, in NN, $r_i(\mathbf{y})$ is written as

$$r_i(\mathbf{y}) = \min_{j \in \{1,\ldots,N_i\}} \|\mathbf{y} - \mathbf{v}_i^{(j)}\|_2. \tag{1}$$

In NFL, $r_i(\mathbf{y})$ is defined as

$$r_i(\mathbf{y}) = \min_{\lambda \in \mathbb{R}, a,b \in \{1,\ldots,N_i\}} \|\mathbf{y} - \lambda \mathbf{v}_i^{(a)} - (1-\lambda)\mathbf{v}_i^{(b)}\|_2. \tag{2}$$

2.2. NS

In NS, the minimum distance $r_i(\mathbf{y})$ is the projection distance from $\mathbf{y}$ to the subspace linearly spanned by all the training samples in the i-th class.

More specifically, given N_i training samples in the i-th class, define

$$\mathbf{V}_i = [\mathbf{v}_i^{(1)}, \ldots, \mathbf{v}_i^{(N_i)}] \tag{3}$$

where $\mathbf{v}_i^{(j)} \in \mathbb{R}^D$ is the j-th training sample and D is the feature dimension.

Note that we assume that the training samples $\mathbf{v}_i^{(1)}, \ldots, \mathbf{v}_i^{(N_i)}$ are linearly independent. Namely, $\mathbf{V}_i \in \mathbb{R}^{D \times N_i}$ is a full rank matrix, satisfying $D \geq N_i$. Henceforth, unless otherwise stated, we assume that the given training samples are linearly independent. This assumption is satisfied in many pattern recognition problems in which the feature space has a high dimension.

Then, in NS, $r_i(\mathbf{y})$ is defined as

$$r_i(\mathbf{y}) = \min_{\boldsymbol{\alpha}_i \in \mathbb{R}^{N_i}} \|\mathbf{y} - \mathbf{V}_i \boldsymbol{\alpha}_i\|_2 \tag{4}$$

where $\boldsymbol{\alpha}_i \doteq [\alpha_i^{(1)}, \ldots, \alpha_i^{(N_i)}]^{\mathrm{T}}$ is the coefficient vector.

If the N_i training samples are linearly independent, the spanned subspace is N_i-dimensional.

After the distances from $\mathbf{y}$ to K classes are obtained, NN, NFL and NS, using the same scheme, determine the class of $\mathbf{y}$ by

$$\text{class}(\mathbf{y}) = \arg\min_{i \in \{1,\ldots,K\}} r_i(\mathbf{y}). \tag{5}$$

3. NM: Nearest Manifold

The NM (Nearest Manifold) classifier is a generalization of the NN, NFL and NS classifiers. It also determines the class of a query sample based on the minimum distance. In NM, the manifold associated with a given class is a topological space such that all the data points (including the observed ones and unobserved ones) of the class are lying on or near to it. Thus, the manifold dimension is actually the intrinsic dimension of the dataset and is usually much less than the dimension of feature space. If a query data sample is near to a data manifold, then it is assigned to the corresponding class.

3.1. Model

For a given a class, we define its universal dataset to be the “conceptual” set containing all the observed and unobserved data of this class. It is assumed that this universal data set forms a manifold in the feature space, and that the dimension of the manifold is much less than the dimension of the feature space.

Given K data manifolds denoted by $\mathcal{M}_1, \ldots, \mathcal{M}_K$, $\mathbf{y}$ is assigned to the class whose data manifold is the nearest to $\mathbf{y}$. More specifically, $r_i(\mathbf{y})$ is written as follows:

$$r_i(\mathbf{y}) = \min_{\mathbf{z} \in \mathcal{M}_i} \|\mathbf{y} - \mathbf{z}\|_2, \quad \forall i = 1, \ldots, K. \tag{6}$$

After that, NMC (Nearest Manifold Classifier) uses Eq. (5) to classify $\mathbf{y}$.

3.2. Classifier design based on the least distance

Although it is difficult to implement the NM classifier primarily due to the difficulty of deducing the K data manifolds from the given training samples, the NM model gives us some clues for designing a good classifier based on the nearest distance.

Given the training sets for each of K classes, a good classifier can be obtained by adding new derived points to the training sets. In NFL, the derived points consist of points on the feature lines of the class. In NS, the derived points consist of the subspaces spanned by the training samples of the class. The solution of NM is to use data manifolds $\mathcal{M}_1, \ldots, \mathcal{M}_K$ to replace the training sets.

From this point of view, NFL and NS are approximations to NM in that the training sets and the derived points approximate the data manifolds. But these approximations are not necessarily the best.

4. Nearest constrained subspace classifier

We propose a novel classifier called the nearest constrained subspace classifier (NCSC), which generalizes NN, NFL and has a close relationship to NS. The proposed classifier is formulated as