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Rapid and brief communications

A Fourier–LDA approach for image recognition

Xiao-Yuan Jing[a], Yuan-Yan Tang[b], David Zhang[c,*]

[a] *Bio-Computing Research Centre and Shenzhen Graduate School, Harbin Institute of Technology, Shenzhen, Guangdong Province, China*
[b] *Department of Computer Science, Hong Kong Baptist University, Kowloon Tong, Hong Kong*
[c] *Department of Computing, Hong Kong Polytechnic University, Kowloon, Hong Kong*

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Abstract

Fourier transform and linear discrimination analysis (LDA) are two commonly used techniques of image processing and recognition. Based on them, we propose a Fourier–LDA approach (FLA) for image recognition. It selects appropriate Fourier frequency bands with favorable linear separability by using a two-dimensional separability judgment. Then it extracts two-dimensional linear discriminative features to perform the classification. Our experimental results on different image data prove that FLA obtains better classification performance than other linear discrimination methods.

Keywords: Fourier transform; Linear discrimination analysis (LDA); Two-dimensional separability judgment; Frequency-band selection; Fourier–LDA approach (FLA)

1. Introduction

Fourier transform is a widely used image processing technique, which is often applied to the enhancement of image description information and visual effect. In this paper, we combine it with the image recognition technique to enhance the image classification information and improve the recognition effect. Linear discrimination analysis (LDA) is a classical linear discrimination technique. Two famous LDA methods—eigenface [1] and Fisherface [2] are presented by Turk et al. and Belhumeur et al., respectively. In recent years, researchers proposed some new ideas to further develop LDA technique. Yu et al. [3] proposed a direct LDA (DLDA) method. Nevertheless, the experiments of DLDA use all the generated discrimination vectors. This is not in accordance with its theory. Jin et al. [4] proposed an uncorrelated optimal discrimination vectors (UODV) method, which requires the discrimination vectors to simultaneously satisfy the Fisher criterion and the constraint of statistical uncorrelation. Liu et al. [5] presented two enhanced FLD (EFM) face recognition models, yet they do not provide a strategy for the automatic selection of principal components. In this paper, we first present a two-dimensional separability judgment that can facilitate the selection of useful Fourier frequency bands for image recognition, because not all the bands are useful in classification. We then propose to carry out image classification using a new linear discrimination approach—Fourier–LDA approach (FLA). The experiments on different image data will provide the comparison of the classification performance of FLA with the current representative linear discrimination methods.

2. Approach description

2.1. Selection of Fourier frequency bands

Suppose that the original image sample set is X, each gray image matrix is sized $M \times N$ and expressed by $f(x, y)$,

* Corresponding author. Tel.: +852 2766 7271; fax: +852 2774 0842.
E-mail address: csdzhang@comp.polyu.edu.hk (D. Zhang).

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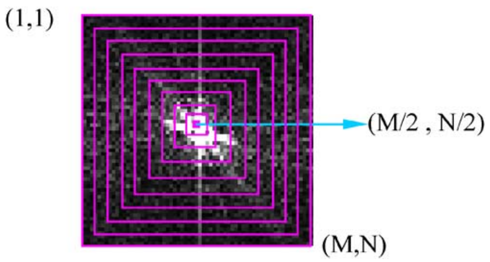

Fig. 1. Illustration of expression way of Fourier frequency bands.

where $1 \leqslant x \leqslant M$, $1 \leqslant y \leqslant M$ and $M \geqslant N$. Assuming there are c known pattern classes $(w_1, w_2, \ldots, w_c)$ in X, where $P_i (i = 1, 2, \ldots, c)$ denotes the priori probability of class w_i. Perform a two-dimensional discrete Fourier transform on each image by

$$F(u, v) = \frac{1}{MN} \sum_{x=1}^{M} \sum_{y=1}^{N} f(x, y) \times \exp\left[-\mathrm{j}\, 2\pi \left(\frac{ux}{M} + \frac{vy}{N}\right)\right], \tag{1}$$

where $\mathrm{j} = \sqrt{-1}$, $\exp()$ denotes the exponential function, and $F(u, v)$ is sized $M \times N$. Let $F(u_0, v_0)$ indicate the zero frequency band. Shift $F(u_0, v_0)$ to the center of image matrix, that is, the point $(M/2, N/2)$. Since the frequency domain is represented by the matrix form, we use a square ring $Ring(k)$ to represent the kth frequency band, where $0 \leqslant k \leqslant M/2$. The four vertexes of $Ring(k)$ are $(u_0 - k, v_0 - k)$, $(u_0 + k, v_0 - k)$, $(u_0 - k, v_0 + k)$ and $(u_0 + k, v_0 + k)$, respectively. So, the kth frequency band denotes

$$F(u, v) \in Ring(k). \tag{2}$$

Different Fourier frequency bands with the above expression way are illustrated in Fig. 1. If we select the kth frequency band, keep the original values of $F(u, v)$, otherwise set the values of $F(u, v)$ to be zero. Which principle should we follow to select the appropriate bands? Here, we propose a two-dimensional separability judgment to evaluate the separability of the frequency band and select the appropriate bands:

(i) Use the kth frequency band:

$$F(u, v) = \begin{cases} Original\ values & \text{if } F(u, v) \in Ring(k), \\ 0 & \text{if } F(u, v) \notin Ring(k) \end{cases} \tag{3}$$

and perform an inverse Fourier transform on the current $F(u, v)$ values as follows:

$$f(x, y) = \frac{1}{MN} \sum_{u=1}^{M} \sum_{v=1}^{N} F(u, v) \times \exp\left[\mathrm{j}\, 2\pi \left(\frac{ux}{M} + \frac{vy}{N}\right)\right]. \tag{4}$$

Thus, for the images in X, we obtain the corresponding band-pass filtered images, which construct a new two-dimensional sample set Y_k. Obviously, Y_k and X have the same values of the number of classes, the number of samples and the priori probabilities. Let A_i $(i = 1, 2, \ldots, c)$ denote a mean value of class w_i and A denote the total mean value of Y_k. Note that A_i and A are in the form of the matrix. With regard to Y_k, the between-class scatter matrix G_b, the within-class scatter matrix G_w are defined as

$$G_b = \sum_{i=1}^{c} P_i \left[(A_i - A)(A_i - A)^{\mathrm{T}}\right], \tag{5}$$

$$G_w = \sum_{i=1}^{c} P_i E\left[(Y_k - A_i)(Y_k - A_i)^{\mathrm{T}}\right]. \tag{6}$$

(ii) We evaluate the separability of Y_k, $J(Y_k)$, using the following judgment:

$$J(Y_k) = \frac{tr(G_b)}{tr(G_w)}, \tag{7}$$

where $tr()$ represents the trace of the matrix. If $tr(G_b) > tr(G_w)$, then we have

$$J(Y_k) > 1. \tag{8}$$

In this situation, according to the Fisher criterion, there is more between-class separable information than within-class scatter information for Y_k. In other words, the corresponding frequency band has good linear separability. Then, for all the frequency bands, we select the bands satisfying Eq. (8), perform the inverse Fourier transform and obtain the band-pass filtered image. Thus, the generated training sample set has favorable total separability. The experiments will show that the total separability value is generally between the minimum and the maximum of $J(Y)_k$. Note that if we only use one frequency band with the maximum of $J(Y)_k$, it is difficult to guarantee that the selected band has good generalization capability in classification because the number of image training samples is always very limited. Therefore, for image recognition, a range of frequency bands should be used.

2.2. FLA

Now, we introduce the FLA discrimination approach:

Step 1: Use the measure introduced in Section 2.1 to select the appropriate frequency bands, where $0 \leqslant k \leqslant M/2$:

$$F(u, v) = \begin{cases} Original\ values & \text{if } F(u, v) \in Ring(i) \text{ and } J(Y)_k > 1, \\ 0 & \text{if } F(u, v) \in Ring(i) \text{ and } J(Y)_k \leqslant 1. \end{cases} \tag{9}$$