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Fast and efficient narrow volume reconstruction from scattered data

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ABSTRACT

We describe a fast and efficient numerical algorithm for the process of three-dimensional narrow volume reconstruction from scattered data in three dimensions. The present study is an extension of previous research [Li et al., Surface embedding narrow volume reconstruction from unorganized points, Comput. Vis. Image Underst. 121 (2014) 100–107]. In the previous work, we modified the original Allen–Cahn equation by multiplying a control function to restrict the evolution within a narrow band around the given surface data set. The key idea of the present work is to perform the computations only on a narrow band around the given surface data set. In this way, we can significantly reduce the storage memory and CPU time. The proposed numerical method, based on operator splitting techniques, can employ a large time step size and exhibits unconditional stability. We perform a number of numerical experiments in order to demonstrate the efficiency of this method.

1. Introduction

In this study, we consider a fast and efficient numerical algorithm for three-dimensional (3D) volume reconstruction from scattered data in three dimensions. Scattered data is defined as a set of data with no specified ordering and connectivity between data points. In practice, the unorganized sample points in $\mathbb{R}^3$ for surface reconstruction can stem from a variety of sources, including medical imagery, laser range scanners, contact probe digitizers, radar and seismic surveys, or mathematical models such as implicit surfaces [1]. Volume reconstruction from scattered data represents an important task. For example, a reverse engineering problem involves reconstructing 3D models from unorganized points that are generated by 3D surface scanning devices [2]. However, volume reconstruction is a challenging problem because point clouds lack ordering information and connectivity, and are usually noisy [3].

Many approaches to surface reconstruction from scattered points exist. Most surface reconstruction methods for point clouds can be classified as either explicit or implicit surface methods, depending on the form of the representation of the surface [4]. In explicit surface representations, the surface location and the geometry are prescribed in an explicit manner. For example, Boissonnat [5] suggested the use of Delaunay triangulations to construct a single connected shape of a point set. This method progressively eliminates tetrahedra from the Delaunay triangulation based on their circumspheres. Typically, in implicit surface reconstruction methods, a signed distance scalar function is constructed on a fixed rectangular grid such that the given scattered data are close to the zero level set of the function. The final 3D shape is the zero isosurface of the signed distance function [3,6–9]. Ye et al. [3] proposed a novel fast method for implicit surface reconstruction from unorganized point clouds. Their algorithm employs a computationally efficient multigrid solver on a narrow band of the level set function that represents the reconstructed surface.

In this study, we focus on implicit representations, and our approach is based on a phase-field model defined by the Allen–Cahn (AC) equation [10]. The AC equation has an intrinsic smoothing effect on interfacial transition layers and the motion by mean curvature. In the level set framework, an explicit time integration scheme is a general choice for the mean curvature flow, which requires small time steps in order to ensure numerical stability. However, for the AC equation a fast and accurate hybrid numerical solver is available [11], which is the main reason why we choose this equation. In our previous work [12], we presented a fast and accurate numerical method for surface embedding narrow volume reconstruction with a fixed distance from an unorganized surface data set. Fig 1(a), (b), (c), and (d) shows a given set of scattered data, its 3D reconstruction, and the cut view and cross view of the reconstructed volume, respectively.

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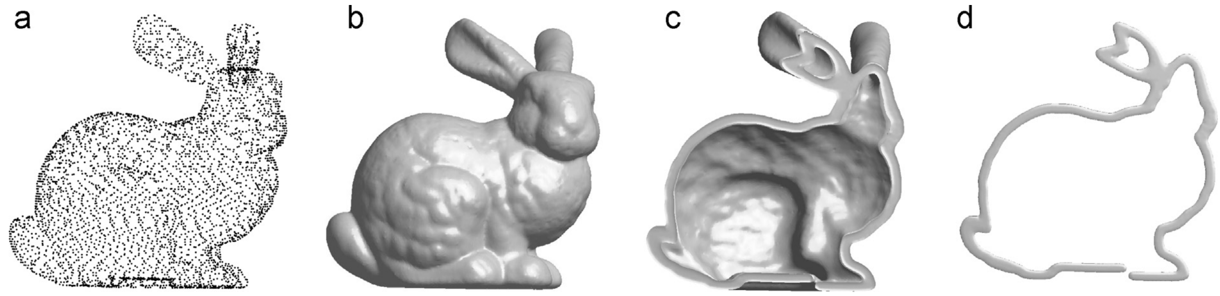

Fig. 1. (a) Point data set, (b) reconstruction, (c) cut view, and (d) cross view of a reconstructed volume.

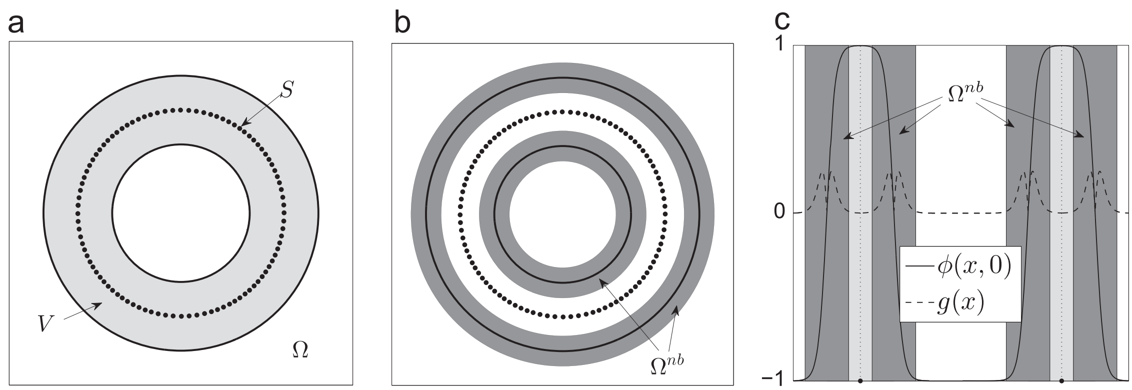

Fig. 2. (a) S is the given scattered data, V is the volume embedding S, and Ω is the global domain containing V. (b) Narrow band domain $\Omega^{nb} = \{\mathbf{x} | \ |\phi(\mathbf{x},0)| < 1-\delta\} \bigcap \Omega$. Here, δ is a small positive value. (c) Sectional view of the new edge stopping function $g(\mathbf{x})$.

Note that our problem is also similar to the offset surface reconstruction problem, which can also be defined as a surface whose points are at a fixed normal distance from a given surface. Many algorithms have been proposed for solving offset surface reconstruction problems. For example, by using an implicit function, Liu and Wang [13] approximated both the zero-level surface and its offset surface. Subsequently, they also developed a fast offset surface generation method via a narrow band signed distance-field [14]. In addition, Chen and Wang proposed thickening operations for converting a surface to a solid [15], and introduced a uniform offsetting model that enables the generation of both enlarged and contracted models from an arbitrary offset distance [16]. Lien [17] and Varadhan and Manocha [18] proposed a method that generates point-based Minkowski sum boundaries. Curless and Levoy developed two important techniques for reconstructing complex and accurate models from scanned objects. The first is spacetime analysis, a ranging method based on analyzing the time evolution of the structured light reflections [19], and the second is a volumetric space carving technique for integrating several data into a single geometric model [20]. In order to obtain high-quality offsets, an adaptive octree-structure was used for distance bounds in [21]. Small and thin features were detected using subdivisional methods [22,23]. For some other methods of offset surface reconstruction, we also refer the reader to [24–30].

The main purpose of the present work is to perform the computations only on a narrow band around the given surface data set, in order to achieve significant reduction in both the storage memory and the CPU time. In addition, the proposed numerical method can use large time step sizes, and exhibits unconditional stability. The proposed numerical scheme has the advantage that the narrow domain can be theoretically defined, and its boundary condition can be defined simply as a Dirichlet boundary condition without loss of accuracy. It should be pointed out that the proposed method is simpler and more efficient than the standard adaptive octree-structure method.

The rest of this paper is organized as follows. In Section 2, we briefly describe the main governing equation. We describe the numerical solution algorithm in Section 3. In Section 4, we perform some characteristic numerical experiments for volume reconstruction. Finally, our conclusions are presented in Section 5.

2. Phase-field method

For scattered surface data points $S = \{\mathbf{X}_p = (X_p, Y_p, \ Z_p) \in \mathbb{R}^3 | p = 1, \ldots, M\}$, where M is the number of data points, we want to reconstruct a uniform narrow volume with a distance l from the given unorganized surface data. In order to find a smooth narrow volume, we presented the following partial differential equation in our previous study [12]:

$$\frac{\partial \phi(\mathbf{x},t)}{\partial t} = g(\mathbf{x})\left(-\frac{F'(\phi(\mathbf{x},t))}{\epsilon^2} + \Delta\phi(\mathbf{x},t)\right), \quad (1)$$

$$\phi(\mathbf{x},0) = \tanh\left(\frac{l-d(\mathbf{x})}{\sqrt{2}\xi}\right), \quad (2)$$

$$g(\mathbf{x}) = 1-\phi^2(\mathbf{x},0), \quad (3)$$

where $\phi \in [-1,1]$ is the order parameter with $\phi = 1$ and $\phi = -1$ inside and outside of the reconstructed narrow volume, respectively. $\phi = 0$ is interpreted as the surface of the volume. $F(\phi) = 0.25(\phi^2-1)^2, \epsilon$ and ξ are positive constants. $d(\mathbf{x})$ is the unsigned distance function from the surface. From (2), we can see that if $d(\mathbf{x}) = l$, then $\phi(\mathbf{x},0) = 0$, which means that the initial guess of ϕ is sufficiently close to the exact solution. If $g(\mathbf{x}) \equiv 1$, then (1) becomes the classical AC equation [10].