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Integral invariants for space motion trajectory matching and recognition

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A B S T R A C T

Motion trajectories provide a key and informative clue in motion characterization of humans, robots and moving objects. In this paper, we propose some new integral invariants for space motion trajectories, which benefit effective motion trajectory matching and recognition. Integral invariants are defined as the line integrals of a class of kernel functions along a motion trajectory. A robust estimation of the integral invariants is formulated based on the blurred segment of noisy discrete curve. Then a non-linear distance of the integral invariants is defined to measure the similarity for trajectory matching and recognition. Such integral invariants, in addition to being invariant to transformation groups, have some desirable properties such as noise insensitivity, computational locality, and uniqueness of representation. Experimental results on trajectory matching and sign recognition show the effectiveness and robustness of the proposed integral invariants in motion trajectory matching and recognition.

1. Introduction

A motion trajectory, which records a sequence of moving positions of a tracked object, provides a compact and representative clue for motion characterization. It has been extensively studied for describing activities, behaviors, and motion patterns in different applications such as learning motion patterns [1,2], human action recognition [6,7], human–robot interaction [3], gesture recognition [4,8], and trajectory retrieval [49]. As these applications suggest, motion trajectories play an important role in determining the contents of video, perceiving similar motion patterns, retrieving actions, and so on. Min et al. [6] employed motion trajectories tracked from some body joints as features input to a discriminant model for classifying human activities. Yang et al. [8] modeled a gesture recognition system using a time-delay neural network, where motion patterns are learned from hand trajectories. Oikonomopoulos et al. [9] also tracked hand trajectories to understand human actions based on the RVM discriminative model [10]. Apart from modeling human actions, motion trajectories of objects of interest are often utilized to build some activity models to understand and retrieve motion patterns in video surveillance [13,49] and for information visualization [5]. Nevertheless, in most related work motion trajectories were often directly used in the raw data form with naïve processing. The raw data rely on the absolute positions of motions in a coordinate system, and are, therefore, ineffective in computation and are sensitive to noise. Not surprisingly, they will change under different viewpoints. Therefore, most space motion trajectory features cannot be captured directly by the raw data.

Shape description has received considerable attention in computer vision for shape matching and classification. In this regard, shape descriptors for describing object contours are closely related to our research. In [21,18], Curvature scale space (CSS) was developed for shape matching. Curvatures of a shape contour at different scales are produced by convolving the shape contour with a series of Gaussian kernels in a coarse to fine manner, where the shape contour is deformed at varying scales, yielding undesirable distortions in the shape. By chain code [11], one can digitize a space curve in terms of relative direction changes of segmented lines. However the relative changes with respect to neighboring lines limited the use of chain code for complex space curves. Using algebraic curve, such as B-spline [24] and Bezier curve [12], a shape contour can be approximated through some key control points. These curve fitting methods show non-uniqueness when the sampling rate of motion trajectories varies or partial occlusions exist in a trajectory, because their approximation accuracies depend on those key control points. Shape context [16] was introduced to capture the histogram bins of neighboring points at each reference point of a curve using a log-polar weight kernel. As a local descriptor, shape context possesses

Abbreviations: MBS, Maximal 3D Blurred Segment; DTW, Dynamic Time Warping; 1-NN, 1-nearest neighbors; FD, Fourier descriptor; II, integral invariants; DI, differential invariants; DII, distance integral invariants; Z, integer set; Z^3, 3-dimensional Euclidean space with only integer sets; I, non-empty interval of real numbers

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rich invariant properties, and is capable of handling occlusions, but it is not the best way to describe a space trajectory due to its coarse distributions captured for a shape. Therefore, even though these shape descriptors have shown good performance in specific applications, they are incapable of fully capturing motion trajectory features in 3D Euclidean space (3D), due to their limited representation capability for simple shape contours (Chain code and shape context), sensitivity to noise (CSS and chain code) and non-uniqueness (B-spline). Several moment invariants for 3D curves under similarity transformations were derived in [47], but they are global descriptors with two limitations: high-order moments are sensitive to noise [48] and they cannot admit invariant to occlusions. In addition, transform functions based on Fourier and Wavelet [22,23] extracted global features from a trajectory, but meanwhile the local features are lost. They are also not stable with respect to noise due to high-order Fourier coefficients involved. All of these shape or curve descriptors were initially constructed for simple planar shapes. They are thus insufficient and non-compact to semantically represent complex 3D space motion trajectories. Therefore, they cannot be extended straightforward to our research. As discussed above, a good descriptor for free-form motion trajectories is expected to be robust to noise and occlusions, and invariant to specific group transformations. Therefore, such a descriptor should necessarily satisfy a number of criteria, some of which are consistent with CSS [21]: uniqueness, invariance, noise resistance, and locality, and it needs to be applicable to both planar and space trajectories.

In this paper, we first review the related literature in invariants, and claim our contributions in invariant representation in Section 2 before proposing the definition and estimation of the new integral invariants in Section 3. In Section 4, we propose the similarity measure that allows warping motion trajectories with various temporal lengths onto each other. We conduct two experiments in Section 5 to show the properties, robustness of the proposed integral invariants through trajectory matching, and their effectiveness in sign recognition. Finally, we conclude the paper in Section 6.

2. Previous work on Invariants and our contributions

Invariants have played an important role for various applications in computer vision ranging from shape representation and matching [17] to object recognition [25] and gesture recognition [19]. Consequently plentiful features that are invariant to specific transformations (affine, similarity, Euclidean) have been investigated in [37,38,40–42]. Two invariant local descriptors related to our research are differential invariants and integral invariants, which have been investigated and put into applications in motion trajectory representation and recognition [32,33]. However, there is a major roadblock that is noise in motion trajectories. Whenever noise is present to affect the spatio-temporal primitives of motion trajectories, differential invariants would be dominated by even small-scale perturbations in that the computation of differential invariants involves high-order derivatives and hence are very sensitive to noise, even though they are approximated [25,27] in terms of joint invariants. Some approaches have tried to overcome this drawback for differential invariants by the introduction of the scale-space smoothing in [39], but a more effective and robust method has so far not been available in principle. There has been much work to attempt deriving integral invariants [14,15,41] based on integral operation. In [41], potentials were proposed to obtain integral invariants for planar shapes via integrating the potentials of the contour curves of shapes, but these integral invariants are global descriptors. Integral invariants for closed planar shapes [14,15] were derived by performing integration of a class of local kernels along the shape boundary represented by a planar curve, where the locality is achieved by restricting integration to local neighborhoods at each point of the curve. Nevertheless, they cannot be extended to represent motion trajectories in 3D case in that the fact of open contours and varying orientations of space trajectories complicates the problems. Consequently, the idea of defining a class of kernel functions along a space trajectory, which admits invariant under group transformations, remains unresolved. Developing an effective, robust invariant representation for space trajectory matching and recognition is a promising topic. In our approach, we will extend the integral invariants for planar closed shapes [14] to define some new integral invariants for free-form space trajectories in 3D Euclidean space.

In this paper, we propose some new integral invariants for space motion trajectories using line integrals of a class of kernel functions along a motion trajectory. Depending on two designed kernel functions, we have two typical integral invariants of transformation groups, the distance and area integral invariants. In this paper, we favor the area integral invariants as our integral invariants in this paper, and define them as the line integrals over a dynamic domain of integration within instant Frenet–Serret frame along a motion trajectory. We then derive a novel estimation formula accordingly to approximate the proposed integral invariants for discrete trajectories based on the Maximal Blurred Segment of discrete curves. In order to match trajectories, we define a distance function of dynamical time warping as the similarity measure between a pair of motion trajectories able to deal with some nonlinear variations including different sampling rates, unequal lengths, and occlusions. Finally, we show the effectiveness, robustness of the integral invariants in motion trajectory matching and recognition.

3. Integral invariants

A space motion trajectory is a sequence of position vectors of a moving object in 3-dimesional Euclidean space, and we denote it with γ: $\boldsymbol{I} \rightarrow \mathbf{R}^3$, parameterized by temporal sequence t :

$$\gamma(t) = \{x(t), y(t), z(t) \mid t \in [a, b]\}, \tag{1}$$

where $[a, b] \in \boldsymbol{I}$ is the time interval and we assume the motion trajectory γ in this paper is a regular curve, i.e., $\|\gamma'(t)\| \neq 0$ at all t. Normally, a space motion trajectory can also be parameterized with respect to arc length s, $\gamma(s) = \{x(s), y(s), z(s)\}$. Note that in practical scenarios motion trajectories are often sampled discretely, and thus t is set to $[1, N] \in \mathbf{Z}$ in this case, where N is the trajectory length (frames). In numerical integration of integral invariants, this discrete temporal parameterization is suitable for modeling the approximation algorithm for numerical integration, whereas in the definition of integral invariants we assume a motion trajectory is a regular curve parameterized with respect to continuous arc length or time sequence.

In this section, we first propose a general definition of integral invariant for a space motion trajectory and then exemplify two specific integral invariants depending on two typical kernels, respectively. The estimation of the area integral invariants for a discrete trajectory is then derived based on the maximal blurred segment of discrete curves. In practical applications, motion trajectories commonly occur in both $\mathbf{R}^2$ and $\mathbf{R}^3$ Euclidean space. Therefore, the proposed integral invariants can be applied to both planar and space trajectories thanks to the property of $\mathbf{R}^2$ being the subgroup of $\mathbf{R}^3$, where the integral invariants for planar trajectories are just the special instances without loss of generality.

3.1. Definition of integral invariant for space trajectory

As addressed in [14,25], it can be deduced that two motion trajectories are equivalent if and only if one can be mapped to another one by a group transformation. Furthermore, they are