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# Geometrical considerations on the directivity of reflected light from a paraboloidal mirror

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**Abstract**

A divergence angle of the light emanated from an optical source composed of a source of light with finite size and a paraboloidal reflector is analyzed on the basis of geometrical considerations. The divergence angle distributions of the reflected light rays from each reflection point on the paraboloidal mirror are numerically calculated as parameters of the shapes of a lamp arc body and a paraboloidal reflector. It has been found from the analyses that the divergence angle can be reduced by prolongation of the optical path length *PO* to the reflection point *P* on the paraboloidal mirror from the focal point *O* of it, at which the center of the extent of the arc is located. A new approach different from conventional one to reduce the divergence angle of the light rays emanated from the optical source is proposed. The new configurations of the optical source composed of a source of light with finite size and a paraboloidal reflector or a spheroidal converging reflector are shown and discussed.


*Keywords:* Directivity; Divergence angle; Geometrical analysis; Paraboloidal reflector; Spheroidal converging reflector; Arc lamp; Source of light; Optical source

## 1. Introduction

In most projectors using liquid crystal (LC) devices or DMDs (Digital Micro-mirror Devices), an optical combination unit composed of a source of light such as an ultra-high pressure (UHP) lamp and a paraboloidal reflector or a spheroidal converging reflector has widely been used as an optical source. In these projectors, optical behaviors of the light propagating through an illumination optics from the optical source have been investigated systematically on the basis of ray-tracing simulation, and an optimum optical system has been designed. The directivity of light emanated from such an optical source greatly depends on the arc gap length of the lamp, and therefore, a shorter-gap lamp has always been demanded. Various efforts to reduce the divergence angle of the light emanated from the optical source have been made in addition to the reduction of arc gap length of the lamp [1]. They are classified roughly into two methods. One method is a transparent optical unit composed of compound aspheric lenses [2], which reduces the divergence angle of the light rays after reflections by a paraboloidal reflector or a spheroidal reflector. Another method is an optimally figured and designed reflector to collect the light rays emanated from a source of light [3–5].

In principle, arrays of laser diodes (LD) or diode-pumped solid-state lasers, including second harmonic generation, are the best sources for projectors such as LC or DMD projector [6,7]. But, these lasers are, now, yet far expensive and too large to use as projector sources. While, the reduction of arc gap length of the UHP lamp involves serious problems such as an explosion or a destruction of the lamp, which have suggested the limitation of ultra-high pressure operation.

In this paper, with the background of these technological situations, the reduction of divergence angle of the light emanated from the optical source composed of a lamp and a paraboloidal reflector or a spheroidal converging reflector is studied. Firstly, the divergence angle of the reflected light from a paraboloidal mirror is analyzed on the basis of geometrical considerations. Especially, the divergence

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angle distributions of the reflected rays from each reflection point on the paraboloidal mirror are derived as parameters of the shapes of a lamp arc body and a paraboloidal reflector. Secondly, according to these analytical results, a new approach different from the conventional ones described above to reduce the divergence angle of the light rays emanated from the optical source is proposed. The new configurations of the optical source composed of a source of light with finite size and a paraboloidal reflector or a spheroidal converging reflector are shown and discussed.

## 2. Analyses

### *2.1. Optical source with a rectilinear shape of arc*

To begin with, as a simple case, we suppose an optical source composed of a rectilinear shape of the arc $Q-Q'$ in length $l$ and a paraboloidal reflector as shown in Fig 1.

In this figure, the parabola, expressed by the equation $y=2(p(x+p))^{1/2}$, shows a cross-section of reflector surface in a paraboloid of revolution around the $x$ axis in the rectangular $x$–$y$–$z$ coordinates. A beam of light emanated from the arc $Q-Q'$ is reflected by the paraboloidal mirror, and the reflected rays at each point $P(x_0,y_0)$ on the paraboloidal mirror are directed toward the positive $x$ axis, which have variable amounts of the divergence angles $\theta-\theta'$ depending on the coordinates $P(x_0,y_0)$. If we define the angles $\theta$, $\alpha$, and $\theta'$ between the $x$ axis, and the lines $PQ$, $PO$ and $PQ'$, respectively, the two visual angles viewing the half arcs $OQ$ and $OQ'$ from the reflection point $P(x_0,y_0)$ become $\theta-\alpha$ and $\alpha-\theta'$, respectively. The equation of parabola having a focal point at the origin $O$ of $x$–$y$ coordinates is expressed by

$$y^2 = 4p(x + p), \tag{1}$$

where $p$ is the focal length that is defined by the distance from the focal point to the vertex of the parabola. The length $PO$ from the reflection point $P(x_0,y_0)$ to the focal point of the parabola is given by

$$PO = (x_0^2 + y_0^2)^{1/2}. \tag{2}$$

From Eqs. (1) and (2), we obtain

$$PO = (x_0^2 + 4px_0 + 4p^2)^{1/2} = x_0 + 2p. \tag{3}$$

Now, drawing a perpendicular line PH to the directrix ($x=-2p$) from the reflection point $P(x_0,y_0)$, we can find that the length $PH$ equals the length $PO$, since $PH=x_0+2p$. Furthermore, we consider two lines $PJ$ and $PJ'$ to the directrix ($x=-2p$) from the reflection point $P(x_0,y_0)$, inclined from the line PH by angles $\theta-\alpha$ and $\alpha-\theta'$, respectively. Here, $J$ and $J'$ are the points of intersection between the line $PJ$ and the line $PJ'$, and the directrix, respectively. And also drawing a line passing through the origin $O$ and perpendicular to the line $PO$, so that, we have two points of intersection $T$ and $T'$ between the perpendicular line, and the production of the line $PQ$ and the line $PQ'$, respectively. Since $PO=$PH and the angle $TPT'$ ($=\theta-\theta'$) equals the angle $JPJ'$ ($=\theta-\theta'$), we easily find that the triangle $PTT'$ is congruence with the triangle $PJJ'$. Then, it can be found that the length $TT'$ equals the length $JJ'$.

As a consequence, we have found through the considerations described above that the reflected ray at the point $P(x_0,y_0)$ on the paraboloidal mirror goes straight on toward the positive direction of the $x$ axis, as if a rectilinear shape of virtual arc $JJ'$ with the same length as the $TT'$, not the $QQ'$,

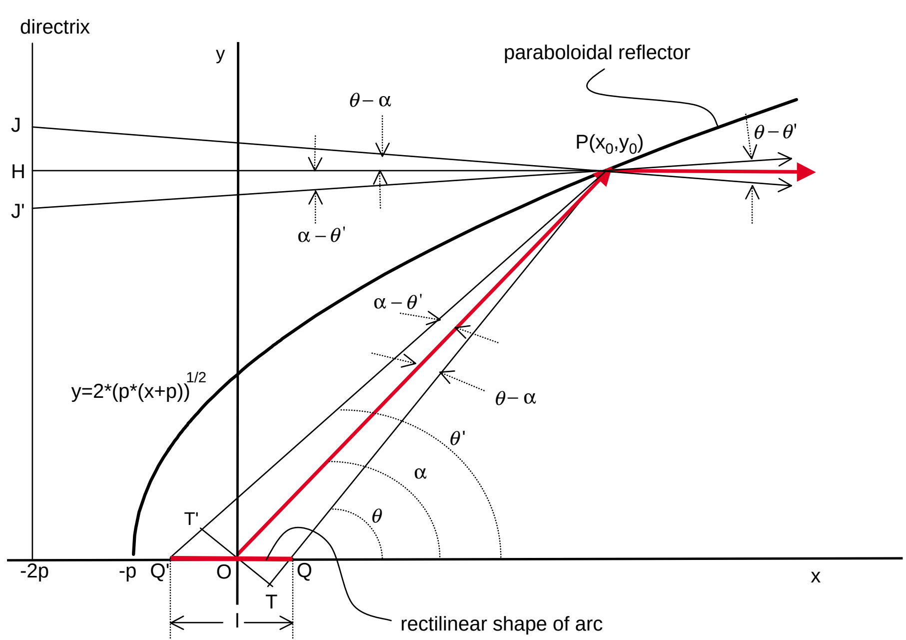


Fig. 1. Geometry of the optical source composed of a rectilinear shape of arc and a paraboloidal reflector.