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Multivariate statistical modeling for image denoising using wavelet transforms

Dongwook Cho*, Tien D. Bui

Department of Computer Science, Concordia University, 1455 De Maisonneuve Blvd. West, Montréal, Qué, Canada H3G 1M8

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Abstract

Recently a variety of efficient image denoising methods using wavelet transforms have been proposed by many researchers. In this paper, we derive the general estimation rule in the wavelet domain to obtain the denoised coefficients from the noisy image based on the multivariate statistical theory. The multivariate distributions of the original clean image can be estimated empirically from a sample image set. We define a parametric multivariate generalized Gaussian distribution (MGGD) model which closely fits the sample distribution. Multivariate model makes it possible to exploit the dependency between the estimated wavelet coefficients and their neighbours or other coefficients in different subbands. Also it can be shown that some of the existing methods based on statistical modeling are subsets of our multivariate approach. Our method could achieve high quality image denoising. Among the existing image denoising methods using the same type of wavelet (Daubechies 8) filter, our results produce the highest peak signal-to-noise ratio (PSNR).

Keywords: Image denoising; Multivariate statistical model; Estimation; Incorporating wavelet coefficients; Wavelet transform

1. Introduction

Various recent works on image denoising using wavelet transforms have shown that wavelet is an efficient tool for noise removal of noisy images [2,3,14,16,18,19]. Since wavelet transform divides an image domain into the low and high frequency domains by its filters, it is natural that some noise which is closely related to the high frequency domain can be removed by killing some small coefficients in the high frequency part.

Donoho proposed a simple thresholding rule which sets all the coefficients smaller than the universal threshold $\sigma\sqrt{2\log n}$ to zero and shrinks the rest of the coefficients by the threshold (*soft-thresholding*) or leaves them without change (*hard-thresholding*) [8]. Donoho's thresholding rule shows that thresholding approach is helpful for denoising and smoothing. However, the universal

*Corresponding author.
E-mail address: d_cho@cs.concordia.ca (D. Cho).

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threshold is a good choice only when the size of input signal n approaches infinity. In practice, when Donoho's threshold is applied to an image, it produces a denoised image which loses many of the details. Fig. 1 shows an example of the denoised images using the universal threshold. Even though the denoised images produce better peak signal-to-noise ratio (PSNR), we can see that they lose a lot of details.

Therefore, a selective image denoising approach which can remove noise yet preserve details must be desirable. As a result, some of the recent papers have proposed adaptive methods to threshold or estimate the coefficients based on the current subband or the local information.

In [9], Donoho and Johnstone presented a thresholding method using Stein's unbiased risk estimator called *SUREShrink*. Unlike the universal threshold, *SUREShrink* estimates a threshold based on all the coefficients in a current subband and achieves much better denoising quality when we apply it to images.

Wavelet coefficients are not strongly correlated, but they still have dependency on each other. So many of the recent works take into account their dependency in order to obtain a proper estimate. Cai and Silverman in [1] proposed a simple and effective approach for signal denoising by incorporating the neighbouring coefficients, called *NeighBlock*. Chen and Bui [4] extended the neighbouring thresholding idea to the multiwavelet case. Mihcak et al. [14] proposed the local variance estimator to get a locally-adaptive shrinkage value. A parent coefficient in the coarser

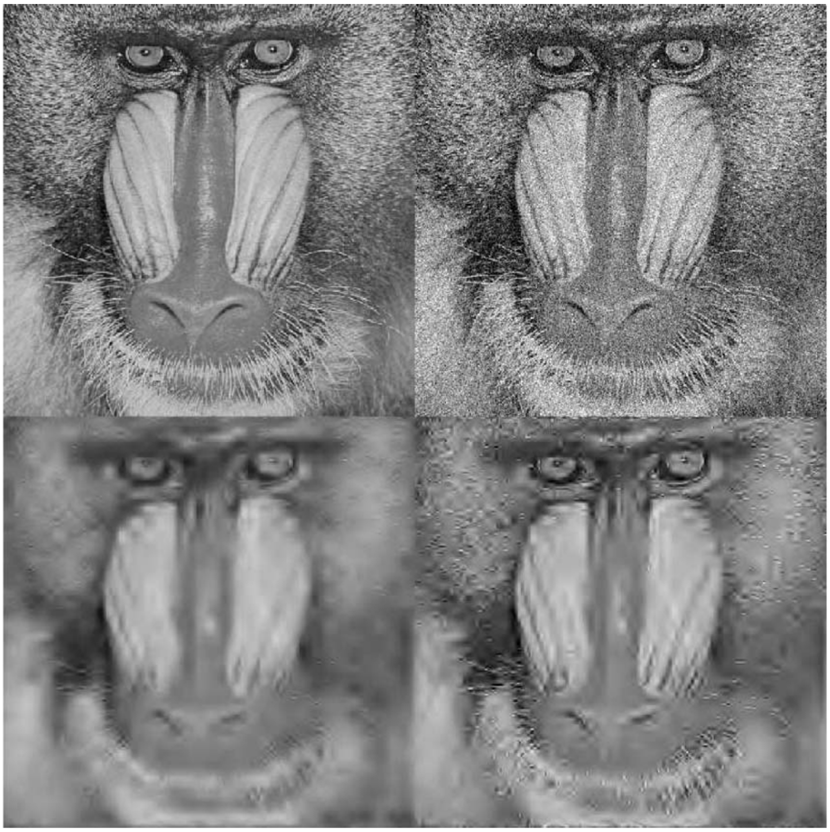

Fig. 1. Denoising example of *Baboon* image applied by Donoho's universal threshold: Original image (top-left), noisy image with $\sigma = 30$ (18.62 dB; top-right), denoised image using soft-threshold (19.80 dB; bottom-left), denoised image using hard-threshold (20.32 dB; bottom-right).