Biomedical Signal Processing and Control 8 (2013) 935–944

Contents lists available at ScienceDirect

Biomedical Signal Processing and Control

journal homepage: www.elsevier.com/locate/bspc

Relating the spectrum of cardiac signals to the spatiotemporal dynamics of cardiac sources

Jesús Requena-Carrión*, Ferney A. Beltrán-Molina, Antonio G. Marques

University Rey Juan Carlos, Fuenlabrada, Madrid 28943, Spain

ARTICLE INFO

Article history:
Received 21 March 2013
Received in revised form 30 August 2013
Accepted 2 September 2013
Available online 9 October 2013

Keywords:
Cardiac spatiotemporal dynamics
Fibrillation
Spectral analysis
Dominant frequency
Lead systems
Measurement sensitivity distribution

ABSTRACT

An increasing number of studies use the spectrum of cardiac signals for analyzing the spatiotemporal dynamics of complex cardiac arrhythmias. However, the relationship between the spectrum of cardiac signals and the spatiotemporal dynamics of the underlying cardiac sources remains to date unclear. In this paper, by following a multivariate signal analysis approach we identify the relationship between the spectrum of cardiac signals, the spatiotemporal dynamics of cardiac sources, and the measurement characteristics of the lead systems. Then, by using analytical methods and computer simulations we analyze the spectrum of cardiac signals measured by idealized lead systems during correlated and uncorrelated spatiotemporal dynamics. Our results show that lead systems can have distorting effects on the spectral envelope of cardiac signals, which depend on the spatial resolution of the lead systems and on the degree of spatiotemporal correlation of the underlying cardiac sources. In addition to this, our results indicate that the spectral features that do not depend on the spectral envelope behave robustly against different choices of lead systems.

1. Introduction

Fibrillation is a complex cardiac arrhythmia whose spatiotemporal characteristics remain poorly understood. Traditionally, fibrillation has been described as random and disorganized, since it induces highly irregular traces in the electrocardiogram (ECG) signal. However, with the application of nonlinear dynamics theory to the investigation of cardiac arrhythmias and the development of optical and electrical mapping techniques, it has been suggested that fibrillation can possess some degree of spatiotemporal regularity [1,2]. This view has led in a natural way to study fibrillation based on the spectrum of cardiac signals such as the ECG and intracardiac electrograms (EGM). Spectral features of cardiac signals have been proposed as experimental indices for detecting ventricular fibrillation (VF) [3], for quantifying the degree of spatiotemporal organization of atrial fibrillation (AF) [4] and for predicting the success of defibrillation shocks [5–7]. Also, intracardiac mapping techniques have been combined with dominant frequency (DF) analysis to study the spatiotemporal characteristics of fibrillation. This method, known as DF mapping, has revealed spatiotemporal regularities during AF in both animal models [8–10] and in patients [11,12] and it is currently regarded as a potential technique to guide AF ablation therapies [13].

Despite the increasing number of studies that use spectral techniques to analyze fibrillation, the meaning of the spectrum of cardiac signals remains to date elusive. Even though individual spectral features of cardiac signals have been linked to spatiotemporal characteristics of cardiac rhythms [14–16], the relationship between the spectrum of cardiac signals and the spatiotemporal characteristics of cardiac rhythms has not been thoroughly investigated. In addition to this, the effects of lead systems on the spectrum of cardiac signals are not well understood, and consequently it is not clear how the spectra of cardiac signals measured by different lead systems relate to one another. The elucidation of the relationship between the spectra of cardiac signals measured by different lead systems is of technical and clinical interest in the context of fibrillation, since it would contribute to the development of novel, improved methods of DF cardiac mapping, such as non-contact intracardiac electrical mapping [17,18] and non-invasive surface ECG mapping [19].

In this paper, we develop a mathematical formalism for investigating, in a systematic way, the spectral manifestation of different cardiac rhythms and the spectral effects of lead systems. By following a multivariate signal analysis approach, we identify the connection between the spectrum of cardiac signals and the spatiotemporal dynamics of the underlying cardiac rhythms. Our formalism allows us to derive theoretical results which are relevant for the analysis and interpretation of the spectrum of cardiac

* Corresponding author at: University Rey Juan Carlos, Department of Signal Theory & Communications, Camino del Molino s/n, Dept III, D207, Fuenlabrada, Madrid 28943, Spain. Tel.: +34 914888463; fax: +34 914887500.
E-mail addresses: jesus.requena@urjc.es (J. Requena-Carrión), ferney.beltran@urjc.es (F.A. Beltrán-Molina), antonio.garcia.marques@urjc.es (A.G. Marques).

1746-8094/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.bspc.2013.09.002

signals, and for devising spectral methods for investigating the spatiotemporal dynamics of cardiac rhythms.

The organization of this paper is as follows. In Section 2 we develop our mathematical formalism and connect the spectrum of cardiac signals to the spatiotemporal dynamics of the underlying cardiac rhythms. Physiologically meaningful cases are studied analytically in Section 3, and in a computer simulation environment in Section 4. Finally, Section 5 contains the conclusions of our investigation and the discussion.

2. Mathematical formalism

In this section we present the mathematical formalism for investigating the spectrum of cardiac signals. Firstly, we introduce the lead-field bioelectric model of cardiac sources and signals. Then, we define the autocorrelation and the spectrum of cardiac sources. Finally, we identify the relationship between: the spectrum of cardiac signals, the spatiotemporal dynamics of cardiac sources and the measurement characteristics of lead systems.

Throughout this paper the following notation is used: $\langle\cdot\rangle_t$ denotes time-average, $\mathcal{F}[\cdot]$ is the Fourier Transform operator, $(*)$ denotes convolution and $\delta(\cdot)$ is the Dirac's delta. We use the following vector definitions: $\mathbf{1} = [1, 1, 1]^T$ and $\mathbf{0} = [0, 0, 0]^T$.

2.1. Bioelectric model

Cardiac sources are the bioelectric processes generated by the heart during contraction. There exist different, equivalent mathematical paradigms to model the activity of cardiac sources, such as the monopole field and the dipole field [20]. In this study, we model cardiac sources as a time-varying dipole field, i.e. as a spatial distribution of time-varying dipoles $\mathbf{J}(v, t) = [J_x(v, t), J_y(v, t), J_z(v, t)]^T$ on a volume V, where v denotes a point located inside V and t denotes the time instant.

The time-varying activity of cardiac sources can be measured by lead systems, producing cardiac signals. Taking the dipole field as our reference description for cardiac sources, we follow a lead-field approach to model cardiac signals [20]. According to the lead-field theory, the cardiac signal $c(t)$ that is induced at a lead system by a dipole field $\mathbf{J}(v, t)$ can be expressed as

$$c(t) = \int_V \mathbf{L}^T(v)\mathbf{J}(v, t)dv, \tag{1}$$

where the vector field $\mathbf{L}(v) = [L_x(v), L_y(v), L_z(v)]^T$ is the measurement sensitivity distribution (MSD) and describes the ability of the lead system to measure cardiac dipoles located at $v \in V$. In words, cardiac signals are a weighted linear combination of the underlying cardiac sources.

2.2. Autocorrelation and spectrum of cardiac sources

The autocorrelation of a cardiac source, $\boldsymbol{\rho}(v, w, \tau)$, $\forall v, w \in V$, is defined as the collection of the cross-correlations between all pairs of dipoles in V. Since cardiac dipoles are vectorial entities, the cross-correlation between two dipoles consists of the cross-correlations between all three components of each dipole [21]. In order to define the autocorrelation of a cardiac source, the average dipole field $\bar{\mathbf{J}}(v)$ needs to be introduced:

$$\bar{\mathbf{J}}(v) = \langle\mathbf{J}(v, t)\rangle_t = [\langle J_x(v, t)\rangle_t, \langle J_y(v, t)\rangle_t, \langle J_z(v, t)\rangle_t]^T. \tag{2}$$

Based on $\bar{\mathbf{J}}(v)$, we define the zero-average dipole field $\mathrm{J}'(v, t) = \mathbf{J}(v, t) - \bar{\mathbf{J}}(v)$, so that $\langle\mathrm{J}'(v, t)\rangle_t = \mathbf{0}$. The cross-correlation matrix between two cardiac dipoles $\mathbf{J}(v, t)$ and $\mathbf{J}(w, t)$, where $v, w \in V$, is then defined as

$$\begin{aligned}\boldsymbol{\rho}(v, w, \tau) &= \langle\mathrm{J}'(v, t + \tau)\mathrm{J}'^T(w, t)\rangle_t \\ &= \begin{pmatrix} \rho_{xx}(v, w, \tau) & \rho_{xy}(v, w, \tau) & \rho_{xz}(v, w, \tau) \\ \rho_{yx}(v, w, \tau) & \rho_{yy}(v, w, \tau) & \rho_{yz}(v, w, \tau) \\ \rho_{zx}(v, w, \tau) & \rho_{zy}(v, w, \tau) & \rho_{zz}(v, w, \tau) \end{pmatrix}.\end{aligned} \tag{3}$$

Therefore, each entry of $\boldsymbol{\rho}(v, w, \tau)$ contains the cross-correlation between one component of $\mathbf{J}(v, t)$ and one component of $\mathbf{J}(w, t)$. For instance, matrix entry $\rho_{zy}(v, w, \tau)$ is $\langle J'_z(v, t + \tau)J'_y(w, t)\rangle_t$. Also, the average power of dipole component $J'_x(v, t)$ is by definition $P_x(v) = \rho_{xx}(v, v, 0)$, and analogous expressions can be obtained for the average power of dipole components $J'_y(v, t)$ and $J'_z(v, t)$.

The spectrum of a cardiac source, $\boldsymbol{\sigma}(v, w, f)$, $\forall v, w \in V$, corresponds to the collection of the cross-spectra between all pairs of dipoles in V, and is defined as

$$\begin{aligned}\boldsymbol{\sigma}(v, w, f) &= \mathcal{F}[\boldsymbol{\rho}(v, w, \tau)] \\ &= \begin{pmatrix} \sigma_{xx}(v, w, f) & \sigma_{xy}(v, w, f) & \sigma_{xz}(v, w, f) \\ \sigma_{yx}(v, w, f) & \sigma_{yy}(v, w, f) & \sigma_{yz}(v, w, f) \\ \sigma_{zx}(v, w, f) & \sigma_{zy}(v, w, f) & \sigma_{zz}(v, w, f) \end{pmatrix},\end{aligned} \tag{4}$$

where the operator $\mathcal{F}[\cdot]$ is applied to $\boldsymbol{\rho}(v, w, \tau)$ on a component-by-component basis. For instance, $\sigma_{zy}(v, w, f)$ is $\mathcal{F}[\rho_{zy}(v, w, \tau)]$.

We also define the *total cross-correlation* $R_J(v, w, \tau)$ between two cardiac dipoles $\mathbf{J}(v, t)$ and $\mathbf{J}(w, t)$ as the sum of the entries of $\boldsymbol{\rho}(v, w, \tau)$ and the *total cross-spectrum* $S_J(v, w, f)$ as the Fourier Transform of $R_J(v, w, \tau)$. Mathematically, they can be expressed as

$$R_J(v, w, \tau) = \mathbf{1}^T\boldsymbol{\rho}(v, w, \tau)\mathbf{1}, \tag{5}$$

$$S_J(v, w, f) = \mathbf{1}^T\boldsymbol{\sigma}(v, w, f)\mathbf{1}. \tag{6}$$

Finally, we define the *normalized* cross-correlation $\rho(v, w, \tau)$ as the matrix of entries

$$\hat{\rho}_{ab}(v, w, \tau) = \frac{\rho_{ab}(v, w, \tau)}{\sqrt{\rho_{aa}(v, v, 0)\rho_{bb}(w, w, 0)}} \tag{7}$$

where $a, b \in \{x, y, z\}$, and the *normalized* total cross-correlation $\hat{R}_J(v, w, \tau)$ as

$$\hat{R}_J(v, w, \tau) = \frac{R_J(v, w, \tau)}{\max_\tau\{R_J(v, w, \tau)\}}. \tag{8}$$

2.3. Autocorrelation and spectrum of cardiac signals

Let $c(t)$ be a cardiac signal measured by applying $\mathbf{L}(v)$ to a cardiac source of autocorrelation $\boldsymbol{\rho}(v, w, \tau)$ and spectrum $\boldsymbol{\sigma}(v, w, f)$ in V. Define $c'(t)$ as the cardiac signal $c(t)$ minus its time-average value $\bar{c} = \langle c(t)\rangle_t$,

$$c'(t) = c(t) - \bar{c}. \tag{9}$$

The autocorrelation function $R_c(\tau)$ of the cardiac signal $c(t)$ is defined as the following average [21]:

$$R_c(\tau) = \langle c'(t + \tau)c'(t)\rangle_t, \tag{10}$$

and its power spectrum $S_c(f)$ is defined as the Fourier Transform of its autocorrelation function,

$$S_c(f) = \mathcal{F}[R_c(\tau)]. \tag{11}$$