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Nonlinear identification of MDOF systems using Volterra series approximation

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ABSTRACT

Most of the practical engineering structures exhibit nonlinearity due to nonlinear dynamic characteristics of structural joints, nonlinear boundary conditions and nonlinear material properties. Meanwhile, the presence of non-linearity in the system can lead to a wide range of structural behavior, for example, jumps, limit cycles, internal resonances, modal coupling, super and sub-harmonic resonances, etc. In this paper, we present a Volterra series approximation approach based on the adaptive filter concept for nonlinear identification of multi-degree of freedom systems, without sacrificing the benefits associated with the traditional Volterra series approach. The effectiveness of the proposed approach is demonstrated using two classical single degrees of freedom systems (breathing crack problem and Duffing Holmes oscillator) and later we extend to multi-degree of freedom systems.

1. Introduction

Volterra series approach is an extension of the convolution integral of linear systems by a series of multi-dimensional convolution integrals. The dynamical properties of the nonlinear system in the Volterra series representation are completely characterized by a sequence of multi-dimensional weighting function called Volterra kernels. These Volterra kernels are the backbone of Volterra series approach in nonlinear analysis and system identification [1,2].

The identification of Volterra kernels of is an ill-posed inverse problem and difficult to determine. A diverse range of time and frequency domain approaches such as cross correlation technique, Wiener series, Kautz filter, Laguerre filter, Kalman filter, wavelets, harmonic probing, NARMAX model, multiwavelet and multi-resolution analysis, neural networks and evolutionary algorithms are reported in the literature for Volterra kernel estimation [3–8]. However the majority of these Volterra kernel identification techniques suffer from limitations such as modeling strong nonlinearities (i.e. subharmonics, chaos) and the convergence of the expansion and their corresponding convergence region [9–11]. While the techniques such as cross correlation and Wiener series require gaussian input excitation, Laguerre filter requires specific broadband amplitude of excitation for accurate kernel estimation. Similarly, the widely used harmonic probing technique requires the harmonic expansion of the system under study need to be derived and is quite laborious for higher order nonlinearities. Even though NARMAX is efficient in modeling subharmonics, it is based on recursive parameter estimation process which is computationally intensive and manual intervention is necessary to eliminate the unnecessary terms.

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In this paper, we present an alternative approach using truncated Volterra series or finite dimensional Volterra series based on adaptive filter theory for nonlinear identification of engineering structures. Even though the adaptive Volterra filter concept has been widely used in the electrical engineering community to identify Volterra kernel coefficients for optical communication systems, noise elimination and image enhancement and restoration [12,13], it has not been so far exploited for nonlinear system identification of civil engineering structures. This method is based on the concept of finite order and memory and provides an alternative form for the computation of Volterra kernels from an ordinary differential equation both in time as well as frequency domain. Numerical Studies are carried out to demonstrate the effectiveness of the proposed adaptive Volterra filter approach for nonlinear system identification. Numerical investigations clearly indicate that the approach has the ability to characterize the type of nonlinearity present in the system and also for parameter estimation even in the presence of measurement noise with a level of good accuracy. The other major advantages of the proposed algorithm are that we can model complex phenomena such as subharmonic oscillations and also we can easily predict the system's response to any arbitrary input only using Volterra kernel coefficients.

2. Volterra series approximation

For a single input single output nonlinear system, with $f(t)$ and $x(t)$ as the input and output respectively, the Volterra series can be expressed as

$$x(t) = \sum_{n=1}^{\infty} x_n(t); \tag{1a}$$

where nth order response component is given by

$$x_n(t) = \underbrace{\int_{-\infty}^{\infty} \dots \dots \int_{-\infty}^{\infty}}_{'n'\,times} h_n(\tau_1, \dots, \tau_n) f(t-\tau_1) \dots f(t-\tau_n) d\tau_1 \dots d\tau_n \tag{1b}$$

The first term in the series is the impulse response function of a linear system in time domain or called as frequency response function (FRF) in frequency domain and $h_n(\tau_1,\dots,\tau_n)$ is the nth order Volterra kernel and its Fourier transform provides the nth order frequency response function (FRF) or Volterra kernel transform as

$$H_n(\omega_1, \dots, \omega_n) = \underbrace{\int_{-\infty}^{\infty} \dots \dots \int_{-\infty}^{\infty}}_{'n'\,\text{times}} h_n(\tau_1, \dots, \tau_n) \prod_{i=1}^{n} e^{-j\omega_i\tau_i} d\tau_1 \dots d\tau_n \tag{2}$$

The discrete-time counterpart of the continuous time domain single degree of freedom (SDOF) given in Eqs. (1a) and (1b) can be written as

$$\begin{aligned} x[n] = h_0 + \sum_{m_1=0}^{M_1} h_1[m_1] f[n-m_1] + \sum_{m_1=0}^{M_1} \sum_{m_2=0}^{M_2} h_2[m_1, m_2] f[n-m_1] f[n-m_2] + \dots \\ + \sum_{m_1=0}^{M_1} \sum_{m_2=0}^{M_2} \sum_{m_p=0}^{Mp} h_p[m_1, m_2, \dots, m_p] f[n-m_1] f[n-m_2] \dots f[n-m_p] + \dots) \end{aligned} \tag{3}$$

where $h_p[m_1, m_2, \dots, m_p]$ is known as the pth order Volterra kernel of the system and $M_1, M_2, \dots, M_p$ are the memory length of the filters. It may be noted that the Volterra kernels are always symmetric in nature.

Similarly, for the multiple input multiple output system, the Volterra series response at the jth location due to excitation at 'i' locations $f_{a_1}(t)$, $f_{a_2}(t)$, ... $f_{a_i}(t)$ can be written as

$$x^{(j)}(t) = \sum_{n=1}^{\infty} x_n^{(j)}(t) \tag{4a}$$

where $x_n^{(j)}(t)$ is the nth-order response component at the jth location and it can be formed as a summation of responses due to all the nth-order components resulting from each individual input force and is expressed as

$$x_n^{(j)}(t) = x_n^{(j:a_1)}(t) + x_n^{(j:a_2)}(t) + \dots \dots \dots + x_n^{(j:a_i)}(t) \tag{4b}$$

Considering a multi-degree of freedom (MDOF) having the forces acting only at 2 locations (i.e. 'a'; and 'b' locations with amplitudes $f_a(t) = A\cos(\omega_1 t)$ and $f_b(t) = B\cos(\omega_2 t)$), the total response of the nonlinear system $x^{(j)}(t)$, at the j^{th} degree of freedom using discrete Volterra series can be written as