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# Nondeterministic wave-based methods for low- and mid-frequency response analysis of acoustic field with limited information

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ABSTRACT

The response of the acoustic field, especially for the mid-frequency response, is very sensitive to uncertainties rising from manufacturing/construction tolerances, aggressive environmental factors and unpredictable excitations. To quantify these uncertainties with limited information effectively, two nondeterministic models (the interval model and the hybrid probability-interval model) are introduced. And then, two corresponding nondeterministic numerical methods are developed for the low- and mid-frequency response analysis of the acoustic field under these two nondeterministic models. The first one is the interval perturbation wave-based method (IPWBM) which is proposed to predict the maximal values of the low- and mid-frequency responses of the acoustic field under the interval model. The second one is the hybrid perturbation wave-based method (HPWBM) which is proposed to predict the maximal values of expectations and standard variances of the low- and mid-frequency responses of the acoustic field under the hybrid probability-interval model. The effectiveness and efficiency of the proposed nondeterministic numerical methods for the low- and mid-frequency response analysis of the acoustic field under the interval model and the hybrid probability-interval model are investigated by a numerical example.


## 1. Introduction

The response analysis of an acoustic field plays an important role in the noise, vibration and harshness optimization of an engineering system. Up to now, there is no effective method for the response analysis of acoustic field in the whole frequency range (20 Hz to 20,000 Hz) which is detectable to the human ear. This whole frequency range can be divided into three parts: the low-frequency range, the mid-frequency range and the high-frequency range. In the low-frequency range, the modals are isolated from each other and the global vibration plays a leading role in the system response. The element-based methods (such as the finite element method and the boundary element method) are the most common numerical techniques for the low-frequency response analysis of the acoustic field [1]. In the high-frequency range, the modals cannot be isolated from each other and the local interactions among the weak coupled subsystems play leading roles in the system response. The statistical energy analysis is the most important technique for the high-frequency response analysis of the acoustic field [2].

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Between the low- and high-frequency ranges, there is a wide mid-frequency range. In the mid-frequency range, even though the modals overlap, they can still be isolated from each other. For the mid-frequency response analysis of the acoustic field, the traditional element-based methods cannot be applied due to their interpolation and pollution errors [3]. Meanwhile, the statistic energy analysis is also unusable as the number of resonant modes in the frequency range of interest is insufficient [2]. However, the mid-frequency response analysis of the acoustic field is very important for the engineering practice. The mid-frequency noise between 200 Hz and 1000 Hz widely exists in engineering. For the mid-frequency response analysis of the acoustic field, many novel element-based methods [4–6] which can achieve the higher computational accuracy and efficiency have been developed. Even though these new element-based methods can raise the analysis frequency, none of them, at least in their current stages of development, can cover the whole mid-frequency range in the most engineering practices. Based on the indirect Trefftz approach, a novel wave-based method (WBM) proposed by Desmet has been applied to the mid-frequency response analysis of the acoustic field [7]. In contrast with the element-based methods, the acoustic domain in WBM is no longer divided into small elements. The response in the entire acoustic domain is expressed in terms of wave functions which are the exact solutions of the governing differential equation. The contributions of wave functions to the response can be obtained by using a weighted residual formulation of the acoustic boundary conditions. Due to its enhanced convergence property, WBM has achieved great successes in a wide range of deterministic acoustic problems, such as the deterministic interior acoustic problem [8,9], the deterministic exterior acoustic problem [10], the deterministic acoustic scattering problem [11,12] and the deterministic structural-acoustic problem [13,14]. Unfortunately, due to the influences of model inaccuracies, multiphase characteristics of materials and unpredictability of environment, uncertainties existing in the acoustic field are unavoidable. The mid-frequency response of the acoustic field is very sensitive to these uncertainties. Without considering these uncertainties, the response of the acoustic field yielded by WBM may be unreliable. Based on this unreliable response, the optimal design of the acoustic field yielded by the deterministic numerical method is questionable. Therefore, it is desired to develop some nondeterministic wave-based methods for the low- and mid-frequency response analysis of the acoustic field with uncertain parameters.

One of the important nondeterministic models to treat with uncertainties existed in the acoustic field is the probabilistic model. In the probabilistic model, the uncertainties are modeled as the probabilistic variables whose precise probability distributions can be obtained. For the frequency response analysis of the acoustic field with probability variables, a lot of probabilistic approaches, such as the field shifting approximation technique [15], the spectral stochastic method [16,17], the inverse mapping technique [18,19] and the hybrid deterministic-statistical analysis method [20–25] have been developed. The main advantage of these probability methods is that the expectation, the variance and even the probability distribution of the response of the acoustic field with probability variables can be yielded. The precondition of the application of probability methods is that the precise probability distributions of uncertain parameters have to be obtained. Unfortunately, in the early stage of design, the information to construct the precise probability distributions of some uncertain parameters may be limited.

The interval model is an important model to treat with the uncertain parameters with limited information. In the interval model, the uncertain parameters are expressed as interval variables whose variational ranges are well-defined. The variational range is a closed bounded set. The interval model has achieved great successes in the static response analysis, the dynamical response analysis and the eigenvalue analysis of structures with interval variables [26–31]. Recently, by combining the finite element method with effective interval approaches, the low-frequency response of the acoustic field with interval variables has been deeply investigated [32–34]. The main shortcoming of the interval method is that only the variational range of the response of the acoustic field can be obtained. The probability distribution of the response within its variational range is missing.

The probability model and the interval model have their own advantages and disadvantages. For some uncertain acoustic fields, the uncertain parameters with and without sufficient information to construct the corresponding probability distributions may exist simultaneously. In this case, a hybrid probability-interval model has been developed. In the hybrid probability-interval model, the uncertain parameters with sufficient information to construct the corresponding probability distributions are treated as probability variables, while the uncertain parameters with limited information are treated as interval variables. The hybrid probability-interval model was firstly applied to the response and reliability analysis of uncertain structures [35–42]. Recently, the hybrid probability-interval model has been extended to the low-frequency response analysis of the acoustic field with both probability variables and interval variables [43,44].

In this paper, two nondeterministic wave-based methods will be proposed for the low- and mid-frequency response analysis of the acoustic field under the interval model or the hybrid probability-interval model. The first one is the interval perturbation wave-based method (IPWBM) in which the interval perturbation technique and the wave-based method are combined in a unified framework. Based on IPWBM, the maximal values of the low- and mid-frequency responses of the acoustic field under the interval model can be evaluated. The second one is the hybrid perturbation wave-based method (HPWBM) in which the hybrid perturbation technique is integrated into the wave-based method. Based on HPWBM, the maximal values of expectations and standard variances of the low- and mid-frequency responses of the acoustic field under the hybrid probability-interval model can be obtained.

The remainder of this paper is organized as follows. In Section 2, the wave-based method for the low- and mid-frequency response analysis of the acoustic field is briefly discussed. In Section 3, IPWBM for the low- and mid-frequency response analysis of the acoustic field under the interval model is derived. In Section 4, HPWBM for the low- and mid-frequency response analysis of the acoustic field under the hybrid probability-interval model is derived. In Section 5, a numerical