Mechanical Systems and Signal Processing 84 (2017) 365–383

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Sensitivity-based model updating for structural damage identification using total variation regularization

Niklas Grip [a,1], Natalia Sabourova [a,*,1,2], Yongming Tu [b,3]

[a] Luleå University of Technology, SE-971 87 Luleå, Sweden
[b] School of Civil Engineering, Southeast University, Nanjing, China

ARTICLE INFO

Article history:
Received 29 February 2016
Received in revised form
4 July 2016
Accepted 8 July 2016

Keywords:
Finite element model updating
Structural damage identification
Total variation regularization
(pseudo) Huber function
Interpolation
Reinforced concrete plate

ABSTRACT

Sensitivity-based Finite Element Model Updating (FEMU) is one of the widely accepted techniques used for damage identification in structures. FEMU can be formulated as a numerical optimization problem and solved iteratively making automatic updating of the unknown model parameters by minimizing the difference between measured and analytical structural properties. However, in the presence of noise in the measurements, the updating results are usually prone to errors. This is mathematically described as instability of the damage identification as an inverse problem. One way to resolve this problem is by using regularization. In this paper, we compare a well established interpolation-based regularization method against methods based on the minimization of the total variation of the unknown model parameters. These are new regularization methods for structural damage identification. We investigate how using Huber and pseudo Huber functions in the definition of total variation affects important properties of the methods. For instance, for well-localized damages the results show a clear advantage of the total variation based regularization in terms of the identified location and severity of damage compared with the interpolation-based solution.

For a practical test of the proposed method we use a reinforced concrete plate. Measurements and analysis were performed first on an undamaged plate, and then repeated after applying four different degrees of damage.

1. Introduction

In this paper, we deal with finite element model updating by the classical iterative sensitivity based method [1,2]. Compared to other finite element model updating methods, the sensitivity based method showed computational efficiency and good sensitivity to small damages [3,2]. Basically, there are two application areas of model updating. In the first place, it is applied in order to increase the reliability of the finite element model and thus, for example, the prediction of the dynamic behavior of the structure under different loads. Another application area is damage identification in structures which is the

* Corresponding author.
E-mail addresses: Niklas.Grip@ltu.se (N. Grip), Natalia.Sabourova@ltu.se (N. Sabourova), tuyongming@seu.edu.cn, yongming.tu@ltu.se (Y. Tu).

[1] The authors were supported by the Swedish Research Council Formas grants (Registration numbers 2007–1430 and 2012–1037) as well as by the Swedish Construction Industry's Organisation for Research and Development (SBUF) Grant 13010.
[2] The author was supported by Elsa and Sven Thysells Foundation for Structural Engineering Studies at Luleå University of Technology.
[3] The author was supported by the National Natural Science Foundation of China (Project number 51378104).

http://dx.doi.org/10.1016/j.ymssp.2016.07.012
0888-3270/

focus of this paper.

In mathematical language, damage identification by finite element model updating is a parameter estimation problem. The finite element model is parameterized by unknown parameters, which are updated by some parameter estimation technique. Usually, these parameters are chosen in such a way that they describe with acceptable precision both the location and the severity of damage. We will therefore call them *damage parameters*. Then, assuming that the model is physically meaningful and thus can accurately represent the behavior of the actual structure, the damage identification problem is reduced to the damage parameter estimation only.

The parameter estimation problems belong to a class of inverse problems, i.e. knowing the model outputs, one needs to obtain the internal model parameters. In the presence of noise in the outputs, which is the case, for example, with vibration tests, the inverse problem becomes ill-posed, i.e. small variations in the outputs lead to unreasonably large variations in the model parameters. Such problems can be solved by using regularization, which is increasingly more often consistently taken into account in the area of structural damage identification [4–9].

In this paper, we investigate a regularization tool for the ill-posed damage identification problem that has its origin in image processing and which is associated with the minimization of the total variation of the unknown parameters distributed over a regular 1D or 2D grid. In short, this approach is to run the damage identification algorithm on all parameters, but with restrictions added via a penalty term that allows for a sharp increase of the parameter value close to a damage, while reducing small local variations of the parameter value in undamaged parts of the structure. We compare then this regularization technique with a rather frequently used interpolation with so-called damage functions introduced in [8]. By using damage or interpolating functions, parameters are updated on a sparse grid and then interpolated to intermediate points. This gives smoothing, but the resolution of the damage identification is reduced to that given by the sparse grid. We show that the total variation based regularization brings the parameter estimation close to the desirable solution and in the case of well-localized damage, it results in a more precise damage identification than the interpolation method.

1.1. Damage parametrization

For the kind of damage identification considered in this paper, a commonly used discrete linear time-invariant model of structural motion is the second order differential equation

$$M\ddot{\boldsymbol{u}}(t) + C\dot{\boldsymbol{u}}(t) + K\boldsymbol{u}(t) = \boldsymbol{f}(t), \tag{1}$$

where the matrices M, C and K are real time-independent square system mass, damping and stiffness matrices of order $d \times d$ with d corresponding to the number of degrees of freedom of the model and $\boldsymbol{u}(t)$ is a time dependent displacement vector with d entries.[4] Dots represent derivatives with respect to time t and $\boldsymbol{f}(t)$ is a vector of external forces. Considering the free vibration case, i.e. $\boldsymbol{f}(t) = \boldsymbol{0}$ and looking for the harmonic solution of Eq. (1) in the form $\boldsymbol{u}(t) = \boldsymbol{\phi}_k e^{j\omega_k t}$, we obtain the following generalized eigenvalue problem

$$\left(-\omega_k^2 M + j\omega_k C + K\right)\boldsymbol{\phi}_k = \boldsymbol{0}. \tag{2}$$

Here, $j = \sqrt{-1}$, $\lambda_k = \omega_k^2 = (2\pi f_k)^2$, and $\boldsymbol{\phi}_k$ are the k^{th} eigenvalue and eigenvector, respectively, whereas f_k is the k^{th} eigenfrequency. From Eq. (2) it is easy to see that changes in system matrices M, C and K cause changes in the modal parameters λ_k and $\boldsymbol{\phi}_k$.

It is common in the literature on the structural damage identification to update system matrices by the substructure matrices [4–6] as follows

$$\begin{aligned} K(\boldsymbol{\alpha}) &= K^0 - \sum_{i=1}^{I_1} \alpha_i K_i, \\ M(\boldsymbol{\beta}) &= M^0 - \sum_{i=1}^{I_2} \beta_i M_i, \\ C(\boldsymbol{\gamma}) &= C^0 - \sum_{i=1}^{I_3} \gamma_i C_i, \end{aligned} \tag{3}$$

where $K(\boldsymbol{\alpha})$, $M(\boldsymbol{\beta})$ and $C(\boldsymbol{\gamma})$ are the improved matrices of the parameterized or corrected model. K_i, M_i and C_i are the constant matrices for the i^{th} element or substructure (group) representing the unknown model property and location and expanded to the size of the global system matrices. α_i, β_i and γ_i are dimensionless updating parameters which can be taken as the negative relative difference of the physical parameter from its initial value, i.e. $\frac{X_i^0 - X_i}{X_i^0}$. This choice of updating parameters comes naturally from the simple isotropic Kachanov–Lemaitre damage theory. In this theory the damage is described by a local reduction of elasticity modulus (see [10, Eq. 1.62]) thereby inducing the reduction of the bending stiffness derived by the elastic beam or plate formulation, namely

[4] Hereafter, we use boldface font for column vectors, which we simply call vectors.