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Hyper-trim shrinkage for denoising of ECG signal

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Abstract

We introduce a new shrinkage scheme, *hyper-trim* that generalizes *hard* and *soft* shrinkage proposed by Donoho and Johnstone (1994). The new adaptive denoising method presented is based on Stein's unbiased risk estimation (SURE) and on a new class of shrinkage function. The proposed new class of shrinkage function has continuous derivative. The shrinkage function is simulated and tested with ECG signals added with standard Gaussian noise using MATLAB. This method gives better mean square error (MSE) performance over conventional wavelet shrinkage methodologies.

Keywords: Wavelet transform; Hyper-trim shrinkage; α-Trimmed filter; Shrinkage function; Firm shrinkage; Non-negative garrote shrinkage

1. Introduction

The electrocardiogram (ECG) obtained by non-invasive technique is a harmless, safe and quick method for diagnostic purposes. The accuracy and the content of diagnostic information extracted from recording requires proper characterization of waveform morphologies that needs better preservation of signal details and higher attenuation of corrupted noise. Recently, wavelet transform has proven to be a useful tool for non-stationary signal analysis. Wavelets provide a flexible prototyping environment that comes with fast computational algorithms. A shrinkage method compares empirical wavelet coefficient

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with a threshold and is set to zero if its magnitude is less than the threshold value [1]. The threshold acts as an oracle, which distinguishes between the significant and insignificant coefficients. Shrinkage of empirical wavelet coefficients works best when the underling set of the true coefficients of f is *sparse*. When overwhelming majority of coefficients of f are small, and the remaining few large ones explain most of the functional form of f.

The wavelet shrinkage here is conceptually inspired by the work of Donoho and Johnstone (1995) as well as the work of Breiman (1995) and Bruce and Gao (1996). Donoho et al. have developed wavelet shrinkage methods for denoising for function estimation [2]. Of these wavelet shrinkage methods, *SureShrink* is an optimized hybrid scale dependent thresholding scheme [5] based on SURE. It combines universal threshold selecting scheme and scale dependent adaptive threshold selection scheme that provides the best estimation results in the sense of l_2 risk when the true function is not known [3]. Asymptotically both hard and soft shrinkage estimates achieve within a factor $\log(n)$ of the ideal performance [1]. The wavelet coefficients at the coarsest scale are left intact, while the coefficients at all the other scales are thresholded via soft shrinkage with the universal thresholding

$$\lambda = \sigma\sqrt{2\log N}, \tag{1}$$

where σ^2 is the noise variance and N is the length of the signal.

The shrinkage functions proposed by Donoho and Johnstone are the hard and the soft shrinkage function:

$$\delta_\lambda^{\mathrm{H}}(x) = \begin{cases} 0, & |x| \leqslant \lambda, \\ x, & |x| > \lambda, \end{cases} \tag{2}$$

$$\delta_\lambda^{\mathrm{S}}(x) = \begin{cases} 0, & |x| \leqslant \lambda, \\ x - \lambda, & x > \lambda, \\ x + \lambda, & x < -\lambda, \end{cases} \tag{3}$$

where $\lambda \in [0, \infty]$ is the threshold.

Note that the derivation of standard soft shrinkage function is not continuous. Both hard and soft shrinkages have advantages and disadvantages. The soft shrinkage estimates tend to have bigger bias, due to the shrinkage of large coefficients. Due to the discontinuities of the shrinkage function, hard shrinkage estimates tend to have bigger variance and can be unstable—that is, sensitive to small changes in the data [4].

To overcome the drawbacks of hard and soft shrinkage, a *non-negative garrote* shrinkage function [4] was first introduced by Breiman (1995) as follows:

$$\delta_\lambda^{\mathrm{G}}(x) = x\bigl[1 - (\lambda/x)^2\bigr]_+ = \begin{cases} 0, & |x| \leqslant \lambda, \\ x - (\lambda^2/x), & |x| > \lambda. \end{cases} \tag{4}$$

The shrinkage function $\delta_\lambda^{\mathrm{G}}(x)$ is continuous and approaches the identity line as $|x|$ gets large. The non-negative garrote shrinkage function provides a good compromise between the hard and the soft shrinkage function. The garrote shrinkage like *firm* shrinkage is less sensitive than hard shrinkage to small fluctuations and less biased than soft shrinkage [4,8]. Breiman showed that non-negative garrote with a consistent lower prediction error than subset selection when the true model has many small nonzero coefficients.