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Fast communication

Viterbi algorithm for chirp-rate and instantaneous frequency estimation

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ABSTRACT

An instance of the Viterbi algorithm has been applied to the cubic phase function and chirp-rate estimation. The proposed algorithm has shown excellent performance for high noise environment. The obtained chirp-rate estimate is used in the instantaneous frequency estimation. The proposed instantaneous frequency estimator gives superior performance with respect to the state-of-the-art techniques for signals with non-linear instantaneous frequency.

1. Introduction

The Viterbi algorithm (VA) for hidden state estimation has found diverse applications in various scientific fields [1,2]. It seems that it is best known in communication systems where it is a common tool for decoding convolution codes. Recently, it has been applied to the Wigner distribution (WD) based instantaneous frequency (IF) estimation [3]. The IF is the most important parameter of non-stationary signals. If it is available or accurately estimated, the estimation of other signal parameters can be performed on dechirped signal in the same fashion like common processing techniques of stationary signals. The time–frequency (TF) representations, of which the WD is a prominent member, are used for development of various non-parametric IF estimation tools [4,5]. Detailed analysis of the TF-based IF estimators is given in [5]. The following sources of errors are identified in the case of the TF-based IF estimators: (a) bias introduced by non-linearity in the IF function; (b) small noise influence that can move peak of the TF representation within the signal auto-term; (c) high noise influence that can move peak of the TF representations outside of the signal auto-term; (d) errors caused by discretization of the TF grid; (e) influence of other signal components (cross-terms), etc. The mentioned instance of the VA has been primarily developed for handling the high noise influence [3]. Originally it has been applied to the WD (referred here as the VA-WD) but it has also been used for other TF representations and for some practical applications [6]. Furthermore, it has been applied on connecting components in the TF plane like in [7]. The proposed algorithm is quite robust to noise influence and the experiments have shown that it is the most robust existing technique for extremely high noise [3]. The proposed technique is able to produce accurate results for linear FM signals up to -10 dB. For signals with non-linear IF the amplitude of the WD decreases, i.e., the WD spreads over TF plane. Position of the maximum is moved from the IF due to the higher order derivatives in signal phase [8] causing the estimation bias. In addition, decreasing of the WD amplitude causes reduced robustness to the noise influence. The VA-WD improves results for such signals

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but not in so spectacular manner as in the case of the linear FM signals.

Recently, O'Shea has proposed the cubic phase function (CPF) for the chirp-rate (CR) estimation [9]. This function is quite similar to the WD having the same order of non-linearity but it is able to estimate the CR (second derivative of the signal phase). In this paper, we have proposed the VA for the CR estimation based on the CPF (VA-CPF for short). This is straightforward extension of the algorithm from [3]. Like in [3], the proposed technique significantly improves estimation of the CR for high noise environments. Here, we have used the obtained CR estimate to perform the IF estimation in the second stage. This technique produces significantly better results for signals with non-linear IF than the original technique from [3] (it will be referred to as the VA-CPF-WD for short since the WD is used in the second step, i.e., in the IF estimation).

The letter is organized as follows. The background related to the IF and CR estimation is given in Section 2. The VA-CPF-WD estimator is proposed in Section 3. Numerical study is presented in Section 4 with conclusions and discussions provided in Section 5.

2. IF and CR estimation

Here we will consider frequency modulated (FM) signal defined as

$$x(t) = A\exp(j\phi(t)). \tag{1}$$

The signal is embedded in Gaussian noise with independent real and imaginary parts $y(t) = x(t)+\nu(t)$, where $E\{\nu(t)\} = 0$ and var $\{\nu(t)\} = \sigma^2$. The IF is defined as the first derivative of the signal phase $\omega(t) = \phi'(t)$ while the second derivative of the signal phase is the CR $\Omega(t) = \phi''(t)$. The IF is extremely important since its knowledge or accurate estimation allows simple estimation of other signal parameters (phase and amplitude), accurate time-varying filtering, etc. The CR is also quite important since among other applications its knowledge can be used for focusing of SAR and ISAR images defocused by various phenomena distorting received radar signals [10,11].

2.1. Wigner distribution

The TF representation are common tool for non-parametric IF estimation. The position of the TF representation maxima in the considered instant is the simplest and the most commonly used TF-based IF estimator. Here, the WD is considered

$$WD(n,k) = \sum_{k=-N/2}^{N/2-1} y(n+k)y^*(n-k)\exp(-j4\pi nk/N), \tag{2}$$

where $y(n) = y(n\Delta t)$, Δt is the sampling rate and N is the number of samples within a considered interval. The IF can be estimated as $\hat{\omega}(n) = \pi\hat{k}(n)/(\Delta t N)$ where

$$\hat{k}(n) = \arg\max_{k} WD(n,k). \tag{3}$$

Note that for non-noisy signal the WD is ideally concentrated on the IF for linear FM signal and it can be expected excellent performance of the WD-based IF estimator. However, for non-linear FM signal amplitude of the WD would decrease with the appearance of inner interferences. These effects reduce performance of the WD as the IF estimators. Commonly reducing the bias (inner interferences) by employing the higher-order TF representations has as a drawback increased influence of noise [12].

2.2. Cubic phase function

The CPF is defined as

$$C(n,\Omega) = \sum_{k=-N/2}^{N/2} y(n+k)y(n-k)\exp(-j\Omega(k\Delta t)^2). \tag{4}$$

One may notice similarity with the WD: slightly different auto-correlation and changed complex exponential. However, the CPF is able to estimate the CR

$$\hat{\Omega}(n) = \arg\max_{\Omega}|C(n,\Omega)|. \tag{5}$$

The CPF is ideally concentrated on the CR for cubic phase signals $\phi(t) = \sum_{i=0}^{3} a_i t^i/i!$, with the IF $\omega(t) = \sum_{i=1}^{3} a_i t^{i-1}/(i-1)!$ and the CR $\Omega(t) = \sum_{i=2}^{3} a_i t^{i-2}/(i-2)! = a_2+a_3 t$. For higher-order phase signals bias and other effect will be observed in the CR estimation [13].

3. Viterbi algorithm

3.1. IF estimation

An instance of the VA is developed for the IF estimation of monocomponent signals. The WD was the TF tool on which the VA has been applied. This estimator can be defined as

$$\hat{k}(n) = \arg\min_{k(n)} p(k(n); n_1, n_2), \tag{6}$$

where $p(k(n);n_1,n_2)$ is the sum of path penalty functions in the TF plane from the instant n_1 to the instant n_2 along a line $k(n)$. Commonly used path penalty functions in the VA framework are designed as logarithms of the corresponding conditional probabilities. However, this model is rather hard to be applied in the TF analysis since the conditional probabilities of TF representation states cannot be determined for general non-parametric model of FM signals and noise in an appropriate or closed form. Then two path penalty functions are used. The first one (denoted with $f()$) assumes that the IF in the considered instant is on one of the largest values of the TF representation (now it is allowed that the IF is not strictly on maximum of the TF representation like in (3)). The second one (denoted as $g()$) assumes that the changes of the IF between consecutive instants are not too large. This can be written as

$$\hat{k}(n) = \arg\min_{k(n)} \left[\sum_{n=n_1}^{n_2-1} g(k(n),k(n+1)) + \sum_{n=n_1}^{n_2} f(WD(n,k(n))) \right]. \tag{7}$$

Functions $f()$ and $g()$ are selected in a semi-intuitive manner. Function $f()$ is formed by sorting the WD values for the considered instant and the maximal value is penalized with 0, the second largest is penalized with value 1, the third one with value 2, etc. This clearly reflects