Signal Processing 91 (2011) 1347–1350

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Fast communication

Unfolding the frequency spectrum for undersampled wideband data ☆

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ARTICLE INFO

Article history:
Received 15 June 2010
Received in revised form
10 December 2010
Accepted 24 December 2010
Available online 31 December 2010

Keywords:
Aliasing
Ambiguities
Frequency spectrum reconstruction
Radio frequency measurement systems
Undersampling
Wideband signals

ABSTRACT

In this letter, we discuss the problem of unfolding the frequency spectrum for undersampled wideband data. The problem is of relevance to state-of-the-art radio frequency measurement systems, which capture repetitive waveform based on a sampling rate that violates the Nyquist constraint. The problem is presented in a compact form by the inclusion of a complex operator called the CN operator. The ease-of-use problem formulation eliminates the ambiguity caused by folded frequency spectra, in particular those with lines standing on multiples of the Nyquist frequency that are captured with erroneous amplitude and phase values.

1. Introduction

Digital signal processing has become a pervasive tool for processing measurements that are taken from the real world. Based on the pioneering work by Cooley and Tukey, processing digital data using the fast Fourier transform (FFT) has made a significant impact on the signal processing community [1]. Sampling strategies for the collection of digital data must fulfill the conditions mentioned in Shannon's sampling theorem, i.e., the sampling must be performed at a rate that is at least twice that of the analog data's bandwidth [2]. However, sparse signals may relax the sampling speed [3]. For radio frequency (RF) applications, down-conversion to an intermediate frequency (IF) is a standard approach for handling RF signals with limited bandwidth, which is IF band-pass sampling. A test setup example is presented in Fig. 1.

The field of RF measurement systems is a hot spot because it is a key player in the development of wireless systems and RF products. With the increasing use of bandwidth in modern wireless communication systems, requirements on RF measurement systems have become tighter as higher sampling rates and larger analog bandwidths are required to digitally process such wideband signals. The bottleneck in performance is in the analog-to-digital conversion process, where higher sampling rate result in limited resolution, leading to a trade-off between resolution and speed [4,5]. There is a need for efficient digital signal-processing methods to reconstruct wideband data from undersampled measurements with high resolution, e.g., for the characterization of the out-of-band nonlinear behavior of radio frequency power amplifiers [6].

In applications where repetitive measurements are available, the requirements on the speed of the ADC can be reduced by increasing the number of measurement sets in different manners. Due to the violation of the

☆ Parts of the research was funded by a research grant from the Ericsson's Research Foundation (FOSTIFT-10:019).

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doi:10.1016/j.sigpro.2010.12.013

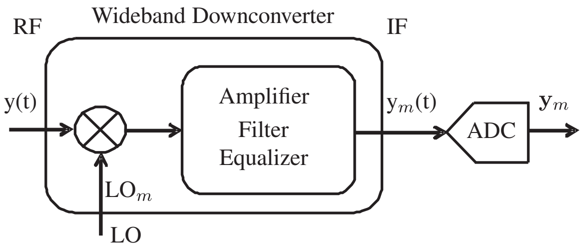

Fig. 1. Signal processing setup. The repetitive wideband analog signal is down-converted M times using different local oscillator frequencies, which results in the *undersampled* data $\mathbf{y}_1,\ldots,\mathbf{y}_M$.

Shannon sampling conditions, aliasing will limit the reproducibility of the undersampled information, which will require the development of state-of-the-art reconstruction methods to overcome the limitations. Nowadays, a main method for such reconstruction is based on time-domain equivalent time sampling (ETS), which is used in many digital oscilloscopes for high frequency acquisition, and standardized in IEEE-STD-1057 [7]. A limitation of ETS is the requirements on the repetition frequency of the waveform [8]. An alternative approach is to work in the frequency domain by estimating the position and complex-valued values of the components of the frequency spectrum.

In this letter, we consider the issue of unfolding the frequency spectrum for undersampled wideband data, noted by its discrete Fourier transform (DFT). Although synthetic sampling is straightforward in theory, it introduces a plurality of practical issues such as the calibration of the measurement setup. From the signal processing point of view, a major issue with FFT-processing is the amplitude/phase ambiguity at the Nyquist frequency due to the violation of the strict inequality in the bandwidth of the analog signal [9,10]. Such phenomena are exemplified in several engineering textbooks [11,12]. Such ambiguity is a show-stopper in the error-free reconstruction of undersampled waveforms with bandwidths that surpass multiples of the Nyquist frequency, i.e., the critical frequencies.

In this letter, we consider the problem of error-free reconstruction of the DFT corresponding to a Nyquist sampled broadband signal, which is based on a set of M measurement sequences that are undersampled by a factor M. The approach utilizes a stepping mechanism in the local oscillator (LO) of the measurement setup shown in Fig. 1. By introducing a complex-notation (CN) operator we present a matrix notation that is suitable for this class of problems. In addition to the compact notation, which has its own right, the results are important for the digital processing of the data, for example, in the calculation of calibration coefficients.

2. Main results

From the theory of aliasing, we know that any frequency component that is higher than half the sampling frequency F_s falls back to the first Nyquist band [i.e., (0, $F_s/2$)]. Frequency components that are in an odd Nyquist band alias back indistinguishably to the first Nyquist band with the same complex form. Frequency components that are in an even band alias back to the first Nyquist band in a mirrored form relative to the Nyquist frequency $F_s/2$ with a conjugate complex form.

Another issue that must be considered is the amplitude and phase ambiguity caused by the critical frequencies which fall back to their DFT counterparts at DC or $F_s/2$. Due to the ambiguity phenomenon, DFT frequency bins at DC and $F_s/2$ are unreliable for the reconstruction of an undersampled signal, and they must be excluded from the calculated DFTs. Although this exclusion violates the inherent structure of the problem, it can be reinforced by combining the CN-operator and the LO stepping mechanism as introduced below.

Consider the test setup shown in Fig. 1 with a repeatable signal applied as the input to the down-converter RF side and one set of measurement data of length N (gathered in the column vector $\mathbf{y}_m$) collected for each setting of the LO. In other words, let the LO span the ordered set in the following equation:

$$\mathrm{LO}_1 > \cdots > \mathrm{LO}_m > \cdots > \mathrm{LO}_M \quad [\mathrm{Hz}] \tag{1}$$

where $m=2,\ldots,M$,

$$\mathrm{LO}_m = \mathrm{LO}_{m-1} - F_s\left(\frac{1}{2}-\frac{1}{N}\right) \quad [\mathrm{Hz}] \tag{2}$$

with initial value LO_1 that is determined below. The set of IFs are given by $F_{\mathrm{IF}}=F_{\mathrm{RF}}-\mathrm{LO}_m$, for $m=1,\ldots,M$. It is assumed that the bandwidth of the RF signal is less than or equal to the IF analog bandwidth of the measurement setup. For the forthcoming discussion, it is assumed that the analog bandwidth is a multiple M of the LO step, i.e., $MF_s(1/2-1/N)$. Further, the measurement system is assumed to be ideal and distortion-free, and down-conversion is performed in a manner such that no aliasing occurs around the zero frequency, i.e., LO_1 is set so that the down-converted RF is properly placed at positive frequencies.

The DFT of each set of data $\mathbf{y}_1,\ldots,\mathbf{y}_M$ is calculated using N-bin FFTs. If we only consider the bins that correspond to strictly positive frequencies, we obtain M DFT column vectors of length $N/2-1$, i.e., $\mathbf{z}_1,\ldots,\mathbf{z}_M$, where the entries $\mathbf{z}_m(k)$ in each of the vectors $\mathbf{z}_m$ are calculated using the DFT in the following equation:

$$\mathbf{z}_m(k) = \sum_{n=0}^{N-1} \mathbf{y}_m(n) e^{-j2\pi kn/N} \tag{3}$$

where k goes from 1 to $N/2-1$. Employing a repeatable stimuli, collected and transformed data from each measurement m are synchronized.

The aim of this work is to construct a vector $\mathbf{u}$ of length $M\,(N/2-1)$ that would been obtained if the IF corresponding to LO_1 was sampled by Nyquist rate $(1-2/N)\;M\;F_s$, with $2M\;(N/2-1)$ samples collected, followed by a DFT retaining the bins corresponding to strictly positive frequencies. For the sake of a compact notation, divide $\mathbf{u}$ into its M sub-vectors, each of length $N/2-1$, according to

$$\mathbf{u} = \begin{pmatrix} \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_M \end{pmatrix}. \tag{4}$$