Signal Processing 93 (2013) 1095–1105

Contents lists available at SciVerse ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Robust sparse channel estimation and equalization in impulsive noise using linear programming

Xue Jiang [a,*], T. Kirubarajan [a], Wen-Jun Zeng [b]

[a] Department of Electrical and Computer Engineering, McMaster University, Hamilton, Ontario, Canada L8S 4L8
[b] Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong

ARTICLE INFO

Article history:
Received 19 March 2012
Received in revised form
26 November 2012
Accepted 27 November 2012
Available online 20 December 2012

Keywords:
Sparse channel estimation
ℓ_1-Regularization
Linear programming
Least absolute deviation
Equalization
Impulsive noise

ABSTRACT

In this paper, an algorithm for sparse channel estimation, called ℓ_1-regularized least-absolutes (ℓ_1-LA), and an algorithm for equalization, called linear least-absolutes (LLA), in non-Gaussian impulsive noise are proposed. The proposed approaches are based on the minimization of the absolute error function, rather than the squared error function. By replacing the standard modulus with the ℓ_1-modulus of complex numbers, the resulting optimization problem can be efficiently solved through linear programming. The selection of an appropriate regularization parameter is also addressed. Numerical results demonstrate that the proposed algorithms, compared with the classical methods, are more robust to impulsive noise and have a superior accuracy.

1. Introduction

In many wireless communication and localization applications, e.g., high definition television (HDTV) terrestrial transmission [1,2], underwater acoustic communication systems [3], and geolocation [4], it is necessary to estimate the multipath propagation channels with a large delay spread but with a small number of non-zero taps. Such channels, an example of which is given in Fig. 3(a), have a long but sparse impulse response. The large delay spread makes estimating the channels of this type a challenge task.

The estimation performance can be improved if the sparse structure of the channels is taken into account. The sparse channel estimation can be viewed as a sparse representation problem. However, finding the sparsest solution, which leads to an ℓ_0-norm minimization problem,[1] is an NP-hard combinatorial optimization problem. To deal with this intractable problem, one appealing method is basis pursuit (BP) [5] or Lasso [6], which replaces the ℓ_0-norm by the ℓ_1-norm. The BP method results in an ℓ_1-regularized least-squares (ℓ_1-LS) problem [7], which can be solved with polynomial-time complexity [8]. The performance of the ℓ_1-minimization based approach is satisfactory when the channel response is sparse [9,10].

Many existing channel estimation methods and equalizers explicitly or implicitly assume that the ambient noise is Gaussian [1,2,9–12]. The ℓ_1-LS based channel estimation approaches and zero-forcing (ZF) equalizer use the squared error function, which is optimal in white Gaussian noise. However, the noise components in practice often exhibit non-Gaussian properties [13]. One important class of non-Gaussian noise frequently encountered in many practical wireless radio systems is the impulsive noise (also referred to as burst noise or outliers) [14–16]. The channel estimation and equalization by minimizing the squared error function is no longer optimal and the performance will degrade in the presence of impulsive noise.

* Corresponding author. Tel.: +1 905 525 9140; fax: +1 905 521 2922.
E-mail address: jiangx5@mcmaster.ca (X. Jiang).

[1] The ℓ_0-norm is defined as the cardinality of the non-zero elements of a vector.

0165-1684/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.sigpro.2012.11.030

Several techniques for sparse representation in impulsive noise are available [17,18]. These sparse signal recovery methods adopt some robust statistics [19] instead of the quadratic residual error function (ℓ_2-norm of the residual vector). In [17], a sparse reconstruction method based on ℓ_1-minimization employing a Lorentzian norm [17] constraint on the residual error is proposed. This method is more robust to outliers by the use of Lorentzian norm instead of the ℓ_2-norm of the residual vector. Despite its good performance in impulsive noise, the Lorentzian norm will lead to a non-convex optimization problem. Finding the global minimum of such a non-convex optimization problem is not easy. In [18], it was proposed to replace the quadratic error function with Huber's penalty function [19] to achieve robustness to impulsive noise. In addition, an iterative procedure is proposed in [18] to solve the resulting minimization of Huber's function with an ℓ_1-regularization term. In each iteration, it requires the solution to an ℓ_1-LS problem. Therefore, the computational complexity of the robust sparse reconstruction algorithm in [18] is high. Moreover, this algorithm for real-valued problems cannot be directly applied to channel estimation and equalization, where complex baseband signals are used.

In this paper, we propose a new channel estimation and equalizer design criterion based on the minimization of the absolute error function with an ℓ_1-norm regularization term. We refer to the proposed channel estimation method and the equalizer as the ℓ_1-regularized least-absolutes (ℓ_1-LA) and linear least-absolutes (LLA), respectively. By using the ℓ_1-modulus of complex numbers instead of the standard modulus, the channel estimation and equalization problem is recast into a linear programming that can be efficiently solved. The ℓ_1-LA based channel estimation algorithm and LLA equalizer are more robust to impulsive noise than the classic methods.

The remainder of this paper is organized as follows. Section 2 introduces the signal model and the channel estimation problem in the presence of impulsive noise. In Section 3, we detail the ℓ_1-LA based sparse channel estimation algorithm and the LLA equalizer that are robust to impulsive noise. A number of numerical simulations are performed to compare the performance of the ℓ_1-LA algorithm and LLA equalizer with those of other representative methods in Section 4. Finally, conclusions are provided in Section 5.

2. Problem formulation

The received discrete complex baseband signal $x(n)$ of a multipath channel in response to the transmitted training sequence $s(n)$ is

$$x(n) = s(n) * h(n) + v(n) \tag{1}$$

with $*$ denoting convolution, $h(n)$ the channel impulse response, and $v(n)$ the additive noise. By modeling the channel response $h(n)$ as a finite impulse response (FIR) filter of order M, the matrix form of (1) is

$$\boldsymbol{x} = \boldsymbol{S}\boldsymbol{h} + \boldsymbol{v}, \tag{2}$$

where $\boldsymbol{x} = [x(0), \ldots, x(N-1)]^T$ and $\boldsymbol{v} = [v(0), \ldots, v(N-1)]^T$ are the vectors of the received signal and noise, $\boldsymbol{h} = [h(0), \ldots, h(M-1)]^T$ is the channel tap coefficients, and the Toeplitz matrix $\boldsymbol{S} \in \mathbb{C}^{N\times M}$ is written as

$$\boldsymbol{S} = \begin{bmatrix} s(0) & s(-1) & \cdots & s(-M+1) \\ s(1) & s(0) & \cdots & s(-M+2) \\ \vdots & \vdots & \ddots & \vdots \\ s(N-1) & s(N-2) & \cdots & s(N-M) \end{bmatrix}. \tag{3}$$

The symbols $s(n)$ with $n<0$ may be obtained from previous decodings in a data stream or can be assumed zero if this is the first received packet. The goal of channel identification is to estimate the channel response $\boldsymbol{h}$ from the training sequence $s(n)$ (matrix $\boldsymbol{S}$) and the received signal $\boldsymbol{x}$, i.e., solving the linear system (2). The linear system in (2) is over-determined (under-complete) since $N > M$.

The previous studies of this problem have focused on the situation where the ambient noise $v(t)$ is additive white Gaussian noise (AWGN) [9,10]. However, in many physical channels, e.g., urban and indoor radio channels [16] and underwater acoustic channels [13], the ambient noise is known through experimental measurements to be decidedly non-Gaussian due to the impulsive nature of man-made electromagnetic interference as well as natural noise [15]. In this paper, we develop linear programming based channel estimation and equalization techniques that are robust to non-Gaussian impulsive noise.

3. Robust channel estimation and equalization using linear programming

3.1. Channel estimation

As pointed above, many channels in practice are sparse. Hence the number of non-zero (dominant) taps K of the channel response $\boldsymbol{h}$ is much smaller than the channel order M, i.e., $K \ll M$. Considering the sparsity of $\boldsymbol{h}$, we aim to find a sparse solution of the linear system $\boldsymbol{x} = \boldsymbol{S}\boldsymbol{h} + \boldsymbol{v}$. As in the BP and Lasso methods, we adopt the ℓ_1-norm as the measurement of sparsity. Then the channel response can be estimated by solving the ℓ_1-regularized least-squares (ℓ_1-LS) problem [7] given by

$$\min_{\boldsymbol{h}} \|\boldsymbol{S}\boldsymbol{h} - \boldsymbol{x}\|^2 + \lambda \|\boldsymbol{h}\|_{\ell_1}, \tag{4}$$

where $\lambda > 0$ is the regularization factor, $\|\cdot\|$ denotes the ℓ_2-norm (i.e., Euclidean norm). Note that the ℓ_1-LS adopts the conventional ℓ_1-norm of a complex-valued vector $\boldsymbol{h}$ that is defined as

$$\|\boldsymbol{h}\|_{\ell_1} = \sum_{i=1}^{M} |h_i| = \sum_{i=1}^{M} \sqrt{(\mathbb{R}\mathrm{e}\, h_i)^2 + (\mathbb{I}\mathrm{m}\, h_i)^2}, \tag{5}$$

where $|\cdot|$ denotes the absolute value of a real number or the modulus of a complex number [7]. $\mathbb{R}\mathrm{e}$ and $\mathbb{I}\mathrm{m}$ denote the real part and the imaginary part of a complex scalar, vector, or matrix, respectively.