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Computationally efficient sparsity-inducing coherence spectrum estimation of complete and non-complete data sets ☆

K. Angelopoulos [a], G.O. Glentis [a,*], A. Jakobsson [b]

[a] Department of Science and Technology of Telecommunications, University of Peloponnese, Tripolis 22100, Greece
[b] Department of Mathematical Statistics, Lund University, P.O. Box 118, SE-221 00 Lund, Sweden

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ABSTRACT

The magnitude squared coherence (MSC) spectrum is an often used frequency-dependent measure for the linear dependency between two stationary processes, and the recent literature contain several contributions on how to form high-resolution data-dependent and adaptive MSC estimators, and on the efficient implementation of such estimators. In this work, we further this development with the presentation of computationally efficient implementations of the recent iterative adaptive approach (IAA) estimator, present a novel sparse learning via iterative minimization (SLIM) algorithm, discuss extensions to two-dimensional data sets, examining both the case of complete data sets and when some of the observations are missing. The algorithms further the recent development of exploiting the estimators' inherently low displacement rank of the necessary products of Toeplitz-like matrices, extending these formulations to the coherence estimation using IAA and SLIM formulations. The performance of the proposed algorithms and implementations are illustrated both with theoretical complexity measures and with numerical simulations.

1. Introduction

The problem of estimating the magnitude squared coherence (MSC) between two or more measured signals is frequently occurring in a wide variety of fields, such as speech processing, time series analysis, geophysics, biomedical engineering, and synthetic aperture radar imaging, wherein one wishes to determine the linear relationship between signals or to determine if a common signal is present in several different measurements. Recently, non-parametric data-adaptive estimation techniques have been exploited to form robust high-resolution MSC estimates [1–5]. In [1,2], it was shown that the one- and two-dimensional (2-D) Capon and APES-based MSC estimators allow for high-resolution MSC estimates, by forming data-adaptive filter banks, with each filter being constrained to pass its center frequency undistorted while suppressing the contribution of all other components. In [3], this work was extended to allow for non-uniformly sampled data by exploiting a formulation based on the recent iterative adaptive approach (IAA) [6]. The IAA-based MSC algorithm, as well as a segmented version termed SIAA-MSC, was there shown to yield reliable estimates even if a large proportion of the measurements are missing. In this paper, we further extend these works by a proposing 1-D and 2-D formulations of the IAA-based MSC estimator, as well as for a novel semi-parametric SLIM-based estimator. The sparse learning via iterative minimization (SLIM) method was introduced in the context of MIMO radar imaging in [7], and can be viewed as a version of the well-known (regularized) FOCUSS

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* Corresponding author. Tel.: +30 2710372202;
fax: +30 2710372161.
E-mail addresses: kaggelop@uop.gr (K. Angelopoulos), gglentis@uop.gr (G.O. Glentis), aj@maths.lth.se (A. Jakobsson).

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algorithm [8], although including also the iterative estimation of the noise variance (see also [9]). Both the IAA and SLIM algorithms have been shown to converge locally [7,10], as well as to yield excellent performance for both complete or incomplete data sets. Regrettably, both algorithms are also computationally cumbersome, and several works have focused on forming various computationally efficient implementations for uniformly and non-uniformly sampled data sequences [11–17]. The presented work may be viewed as a continuation of our recently proposed efficient implementation of the Capon- and APES-based MSC estimators [18], wherein we combined earlier efforts in forming computationally efficient implementations of these spectral estimators [19–26] with the inherently low displacement rank of the estimators' products of Toeplitz-like matrices, thereby allowing for the development of appropriate Gohberg–Semencul (GS) representations of these matrices. The resulting implementation was found to be several orders of magnitude more efficient than the straightforward implementations. Here, building on this work, we extend the IAA- and proposed SLIM-based MSC estimators in a similar way. The paper is organized as follows: in the following section, we briefly review data-adaptive MSC estimation, comparing the formulations of the earlier introduced Capon-, APES-, and IAA-based MSC estimators, as well as introduce a novel SLIM-based MSC estimator. Then, in Section 3, we recall formulations of the MSC estimators using trigonometric polynomials, and then, in Section 4, introduce the efficient implementations of the IAA- and SLIM-based estimators using appropriate GS representations for the necessary products of Toeplitz-like matrices. In Section 5, we proceed to discuss the case of incomplete data sets, followed by the extensions to 2-D formulations of the estimators in Section 6. Section 7 contains a study of the performance of the discussed estimators and implementations. Finally, we conclude on the work in Section 8.

2. Data-adaptive MSC estimation

The MSC spectrum, $\gamma^2_{x_1x_2}(\omega)$, of two stationary complex valued signals, $x_1(n)$ and $x_2(n)$, for $n=0,1,\ldots,N-1$, is defined as (see, e.g., [27–29])

$$\gamma^2_{x_1x_2}(\omega)=\frac{|S_{x_1x_2}(\omega)|^2}{S_{x_1}(\omega)S_{x_2}(\omega)} \tag{1}$$

where $S_{x_1}(\omega)$ and $S_{x_2}(\omega)$ denote the (auto) spectra of the signals $x_1(n)$ and $x_2(n)$, respectively, whereas $S_{x_1x_2}(\omega)$ denotes the cross-spectrum between these two signals. The Capon-, APES-, and IAA-based MSC estimates are formed using the matched filter bank framework (see also [29,30]). Let $\mathbf{h}_N^{(i)}\in C^{N\times 1}$ denote a narrowband data dependent finite impulse response (FIR) filter centered at a generic frequency $\omega\in(-\pi,\pi]$, and form the signals of interest into $N\times 1$ subvectors:

$$\mathbf{x}_N^{(i)}=[x_i(0)\;\; x_i(1)\;\;\ldots\;\; x_i(N-1)]^T \tag{2}$$

where $i=1$ or 2 for the respective signal, and where $(\cdot)^T$ denotes the transpose. As the filters are narrowband, aiming to only pass the generic frequency ω undistorted whereas the contribution from all other frequencies are minimized, the matched filter bank spectral estimate at frequency ω is found as the power of the filtered signal, i.e.,

$$S_{x_i}(\omega)\approx \mathbf{h}_N^{(i)H}\mathcal{R}_N^{(i)}\mathbf{h}_N^{(i)} \tag{3}$$

where $\mathcal{R}_N^{(i)}$ represents the signal's covariance matrix, defined as

$$\mathcal{R}_N^{(i)}=E\{\mathbf{x}_N^{(i)}\mathbf{x}_N^{(i)H}\} \tag{4}$$

with $i=1$ or 2 for the respective signal, where $E\{\cdot\}$ denote the expectation and $(\cdot)^H$ the conjugate transpose, respectively, and where $\mathbf{h}_N^{(i)}$ is a data dependent narrow band filter formed such that

$$\mathbf{h}_N^{(i)}=\underset{\mathbf{h}_N^{(i)}}{\arg\min} S_{x_i}(\omega)\quad \text{s.t.}\quad \mathbf{h}_N^{(i)H}\mathbf{f}_N(\omega)=1 \tag{5}$$

where

$$\mathbf{f}_N(\omega)=[1\;\; e^{\jmath\omega}\;\;\ldots\;\; e^{\jmath(N-1)\omega}]^T \tag{6}$$

is the frequency steering vector. Minimization of (5) with respect to the unknown parameters vector results in a data adaptive and frequency dependent optimal filter of the form:

$$\mathbf{h}_N^{(i)}=\frac{[\mathcal{R}_N^{(i)}]^{-1}\mathbf{f}_N(\omega)}{\mathbf{f}_N^H(\omega)[\mathcal{R}_N^{(i)}]^{-1}\mathbf{f}_N(\omega)} \tag{7}$$

The cross-spectral density needed to form (1) is estimated as

$$S_{x_1x_2}(\omega)\approx \mathbf{h}_N^{(1)H}\mathcal{R}_N^{(12)}\mathbf{h}_N^{(2)} \tag{8}$$

with $\mathcal{R}_N^{(12)}$ denoting the cross-covariance matrix, defined as

$$\mathcal{R}_N^{(12)}=E\{\mathbf{x}_N^{(1)}\mathbf{x}_N^{(2)H}\} \tag{9}$$

Combining (1), (3), (7) and (8), one obtains the Capon-based MSC estimator [1,4]

$$\gamma^2_{x_1x_2}(\omega)=\frac{|\mathbf{f}_N^H(\omega)\mathcal{P}_N^{(12)}\mathbf{f}_N(\omega)|^2}{\prod_{i=1}^2\mathbf{f}_N^H(\omega)[\mathcal{R}_N^{(i)}]^{-1}\mathbf{f}_N(\omega)} \tag{10}$$

where

$$\mathcal{P}_N\triangleq[\mathcal{R}_N^{(1)}]^{-1}\mathcal{R}_N^{(12)}[\mathcal{R}_N^{(2)}]^{-1} \tag{11}$$

2.1. IAA-based MSC estimation

As shown in [1,2], the Capon- and APES-based MSC estimates result from two different design choices for the narrowband filters and use the standard time based averages approximation in place of estimates of the auto and cross correlation matrices. The IAA-based algorithm instead forms the covariance matrices as the sum of the spectral contribution from all possible frequency grid points, essentially viewing that data as a sum of sinusoids, with the number of sinusoids being equal to the size of the frequency grid. Clearly, this is not possible without knowing the amplitudes of all the sinusoids, and, as a result, the estimates are formed using an iterative scheme. Following [3], the data covariance matrices are