Maximum likelihood estimation of DOD and DOA for bistatic MIMO radar

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1. Introduction

Multiple input multiple output (MIMO) radar has become a hot topic which attracts great interests in recent years (see, e.g., [1–3] and the references therein). Compared with traditional phased array radar, which transmits multiple coherent waveforms, MIMO radar offers enhanced performance through waveform diversity and careful radar configurations [4–7]. Currently, MIMO radar can be classified into several types, the first of which is called statistical MIMO radar, i.e., MIMO radar with widely separated antennas. Since different aspects of the target can be viewed simultaneously by statistical MIMO radar, diversity gain and high resolution capabilities can be obtained [8,9]. The second type is the monostatic colocated MIMO radar in which the transmitters and receivers are close enough so that the direction of departure (DOD) of the target and the corresponding direction of arrival (DOA) are identical. In [5,6,10], several advantages of monostatic colocated MIMO radar, including better parameter identifiability, lower-velocity target detection and more flexible transmit beampattern design, have been demonstrated. Slightly different from monostatic colocated MIMO radar, the transmitters and receivers of bistatic MIMO radar are separated away while the transmit elements as well as receive elements are all colocated, respectively. Therefore, the DOD and DOA of a given target are distinct.

In bistatic MIMO radar, the estimation of target DOD and DOA is of much importance and has been extensively discussed recently (see, e.g., [11–17] and the references therein). In [11], a two-dimensional (2D) Capon method was used to estimate the target DOA and DOD. However, it is required to perform exhaustive 2D search to obtain high resolution angle estimations. In [12], estimation of signal parameters via rotational invariance technique (ESPRIT) was applied to the joint estimation of target DOD and DOA, in which both transmit and receive array are assumed to be uniform linear arrays (ULA). Lately, the ESPRIT algorithm proposed in [12] was further improved in [13] where the DOD and DOA are automatically paired. In [14], close form solution for DOD and DOA estimation with ESPRIT was derived, in which spatially colored noise...
can be canceled with three-transmitter configuration. To eliminate the limitation on the number of transmitters in \cite{14}, singular value decomposition (SVD) of the cross correlation matrix was utilized in \cite{16}. In \cite{15}, 2D multiple signal classification (MUSIC) and a double polynomial root finding procedure are proposed for estimating the target DOD and DOA. In \cite{17}, a new method for alleviating the computation burden of 2D MUSIC method was proposed. The target DOD and DOA can be obtained by one-dimensional (1D) search. Moreover, since the method does not impose any constraint on the array configuration, it might be applied to nonuniform linear arrays.

Among these methods, most of them belong to the subspace algorithms, in which it is required the targets are uncorrelated and the signal space can be correctly split from the noise subspace. Thus we need adequate independent and identically distributed (i.i.d.) samples and enough high target signal to noise ratio (SNR) to correctly estimate the corresponding DOA. Typically, multiple i.i.d. samples are used to estimate the target DOD and DOA for bistatic MIMO radar, which has not been discussed yet, to the best of the authors’ knowledge. After formulating the MLE problem of DOD and DOA, we found that the MLE is related to a high-dimensional nonlinear optimization problem, which is typically computationally prohibitive. To solve the MLE problem efficiently, we propose alternating projection (AP) technique to reduce the computation load, by which the original global search in multiple dimensions is replaced by multiple 1D optimization problems. Moreover, a method for adaptively determining the search space in the 1D global optimization is also proposed, which can accelerate the convergence and improve the estimation accuracy of the proposed algorithm.

The remainder of this paper is organized as follows. In Section 2, we present the signal model for the bistatic MIMO radar, followed by a comparison with the monostatic MIMO radar and a preliminary discussion on the localization accuracy of the bistatic MIMO radar in Section 3. Then we derive a compact expression of the CRB for the unbiased estimation of the target DOD and DOA in Section 4. In Section 5, the maximum likelihood estimator of the target DOD and DOA for the bistatic MIMO radar is derived. To solve the high-dimensional nonlinear optimization problem in the MLE efficiently, we propose the AP algorithm in Section 6. Numerical simulation is conducted in Section 7 to illustrate the performance of the proposed algorithm. Finally, we draw the conclusion in Section 8.

**Notation:** Throughout this paper, \( \mathbb{R} \) and \( \mathbb{C} \) denote the set of real and complex numbers. \( \mathbb{R}^{m \times n} \) and \( \mathbb{C}^{m \times n} \) are the sets of matrices of size \( m \times n \) with entries from \( \mathbb{R} \) and \( \mathbb{C} \), respectively. Matrices are denoted by bold italic capital letters, and vectors are denoted by bold italic lowercase letters. Superscript \( (\cdot)^T \), \( (\cdot)^* \) and \( (\cdot)^H \) denote transpose, conjugate, and conjugate transpose, respectively. \( \det(\cdot) \) and \( \text{tr}(\cdot) \) represent determinant and trace of a matrix, respectively. We use \( I_M \) to denote an identity matrix of size \( M \times M \). The complex Gaussian distribution with mean \( m \) and variance \( \Sigma \) is denoted by \( \mathcal{CN}(m, \Sigma) \).

### 2. Signal model

Consider a bistatic MIMO radar with \( N_t \) transmit elements and \( N_r \) receive elements, with a system layout described in Fig. 1. For simplicity, we assume that both of them are linear arrays. Assume that \( N_t \) ideally orthogonal waveforms are transmitted and denote them by \( s_1(t), \ldots, s_{N_t}(t) \), respectively. Then the signal that reaches the target can be written as

\[
a_i^T(\theta_i)S(t),
\]

where \( a_i(\theta_i) \in \mathbb{C}^{N_r \times 1} \) is the corresponding transmit steering vector of the target with DOD \( \theta_i \) and \( S(t) = [s_{1}(t), \ldots, s_{N_t}(t)]^T \in \mathbb{C}^{N_t \times 1} \).

Assume \( P \) targets exist in the range bin of interest, with DOD \( \theta_1, \ldots, \theta_P \) and the corresponding DOA \( \phi_1, \ldots, \phi_P \), respectively. Thus the received signal can be written as

\[
r(t) = \sum_{i=1}^{P} \beta_i a_i(\phi_i) a_i^T(\theta_i) S(t) + n_1(t),
\]

where \( \beta_i \) is the complex amplitude of the \( i \)th target, \( a_i(\phi_i) \in \mathbb{C}^{N_r \times 1} \) is the receive steering vector of \( \phi_i \) and \( n_1(t) \in \mathbb{C}^{N_r \times 1} \) is the noise in the receiver.

Therefore, the output of \( r(t) \) after matched filtering and vectorization can be written as

\[
y = \sum_{i=1}^{P} \beta_i a_i(\theta_i) \otimes a_i(\phi_i) + n = A\beta + n,
\]

where \( \otimes \) denotes the Kronecker product, \( n \in \mathbb{C}^{N_t N_r \times 1} \) is the output of the noise, \( A = [a_1(\theta_1) \otimes a_1(\phi_1), \ldots, a_P(\theta_P) \otimes a_P(\phi_P)] \in \mathbb{C}^{N_t N_r \times P} \) is the transmit–receive array manifold and \( \beta = [\beta_1, \ldots, \beta_P]^T \in \mathbb{C}^{P \times 1} \).

Typically, multiple i.i.d. samples are used to estimate \( \{\theta_i, \phi_i\}, i = 1, \ldots, P \), and the corresponding signal model with multiple snapshots can be written as

\[
y(k) = A\beta(k) + n(k), \quad k = 1, \ldots, K,
\]

where \( K \) is the number of snapshots and \( y(k) \) is the \( k \)th sample.

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Fig. 1. System layout of the bistatic MIMO radar.