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Fast communication

A real-time time-frequency based instantaneous frequency estimator

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ABSTRACT

Commonly used time-frequency representations, like the short-time Fourier transform, the Wigner distribution and the higher order polynomial distributions, estimate the instantaneous frequency at the middle of the time-interval used in the analysis. For real time applications, like for example in radar signal processing, where the target parameters are estimated in the same way as the instantaneous frequency, the delay of a half of the considered interval may be unacceptable. Here, we propose a distribution that inherently estimates the instantaneous frequency at the end of the considered time interval. With a presented procedure for on-line implementation it can outperform other time-frequency representations for real time instantaneous frequency estimation.

1. Introduction

Estimation of the instantaneous frequency (IF) is of great importance in many applications. For example, in radar signal processing it produces information about target's range and cross-range position. Many IF estimators are based on the time-frequency representations [1–7]. Most commonly used are the short time Fourier transform (STFT), the Wigner distribution (WD) and other quadratic reduced interference distributions, defined by the Cohen class. Higher order spectra and distributions are introduced in order to improve the IF estimation, as well. In most of them the IF estimate is obtained for the mid point of the lag interval. Therefore, the IF estimate is delayed for a half of the lag interval. In radars, for example, it means that the estimate of target data is done with a delay corresponding to the half of lag interval (coherence integration time—CIT). This delay can be significant and unacceptable in many cases. Some efforts have been made in the IF extrapolation, in order to deal with this problem.

In this paper we will present a distribution that inherently estimates the IF value at the ending point of the lag window. The distribution preserves property of the WD that it is fully concentrated if the phase variations of signal are up to the quadratic order. Bias and variance analysis of the proposed IF estimator, in the case of signals with non-linear IF, are done. Simulations show the efficiency of the presented distribution, when the current instant for the IF estimate is considered as relevant, rather than the one delayed for half of the lag interval. A procedure for reduced cross-terms realization of the proposed distribution is described. This procedure can be used in the multicomponent signals analysis. It improves performance in the case of high noise, as well.

2. Definition

In recent research, an interest to the highly concentrated distributions has been again increased. For a frequency modulated signal $x(t) = A\exp(j\varphi(t))$ the model

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of a fully concentrated distribution, along the IF

$$TF(t,\omega)=2\pi A^2\delta(\omega-\varphi'(t))=FT_\tau[A^2e^{j\varphi'(t)\tau}] \quad (1)$$

is set again as the basic reference model in sparse time-frequency signal processing (and other recent developments). Assume that the phase function could be considered as quadratic, within the lag time interval. Our goal here is to find the simplest distribution, that is fully concentrated at $\varphi'(t)$, at the current instant t. In contrast to the other distributions derived in a similar way, here we impose the constraint that only the past signal values are available for calculation. This kind of constraint is especially important in real time applications where an IF estimation, delayed by a half of the lag interval, may not be acceptable. In general, time-frequency distributions are defined as the Fourier transform (FT) of generalized local-autocorrelation functions. Here, in the definition of the local auto-correlation function, we use only the current and past instants as

$$r(t,\tau)=e^{j[a_0\varphi(t)+a_1\varphi(t-\tau/2)+a_2\varphi(t-\tau)]}=e^{j\varphi'(t)\tau}. \quad (2)$$

Without loss of generality, a signal normalized in amplitude $A=1$ is used. We also assume that the lag values are of the form τ and $\tau/2$, so that they can be sampled with sampling interval corresponding to the WD calculation, i.e., with $\Delta t/2$. Therefore, an interpolation of discrete signal is not required. After an expansion of the phase functions $\varphi(t-\tau/2)$ and $\varphi(t-\tau)$ into a Taylor series around t, the coefficients should satisfy the following system: $a_0+a_1+a_2=0$; $-a_1/2-a_2=1$ and $a_1/8+a_2/2=0$. It produces $a_0=3$, $a_1=-4$ and $a_2=1$, with

$$r(t,\tau)=x^3(t)x^{*4}\left(t-\frac{\tau}{2}\right)x(t-\tau). \quad (3)$$

A time-frequency representation (TFR) of a continuous signal $x(t)$, then, is

$$D(t,\omega)=\int_0^\infty x^3(t)x(t-\tau)x^{*4}\left(t-\frac{\tau}{2}\right)e^{-j\omega\tau}\,d\tau. \quad (4)$$

Integration is from 0 to ∞ since this distribution is defined to be causal. Notation $x^{*n}(t)$ means the n-th power of complex conjugate of a signal. Since the causal from is used, the representation of a signal $x(t)=A\exp(j\varphi(t))$, with $\varphi'''(t)$ being negligible, is

$$D(t,\omega)=\pi A^2\delta(\omega-\varphi'(t))+\frac{j2\omega}{\omega-\varphi'(t)} \quad (5)$$

with second term, which does not change the nature of distribution begin concentrated at $\omega=\phi'(t)$.

Its pseudo form reads

$$D(t,\omega)=\int_0^h w(\tau)x^3(t)x(t-\tau)x^{*4}\left(t-\frac{\tau}{2}\right)e^{-j\omega\tau}\,d\tau. \quad (6)$$

The past time interval $[t-h,t]$ is used for $D(t,\omega)$ calculation. A lag window is denoted by $w(\tau)$ for $\tau\in[0,h]$. Note that the pseudo WD definition, by using signal values from the same interval, would be of the form

$$WD\left(t-\frac{h}{2},\omega\right)=\int_0^h w(\tau)x\left(t-\frac{h}{2}+\frac{\tau}{2}\right)x^*\left(t-\frac{h}{2}-\frac{\tau}{2}\right)e^{-j\omega\tau}\,d\tau. \quad (7)$$

In order to analyze performance of (6) as an IF estimator, let us consider a single component frequency modulated signal

$$x(t)=A(t)e^{j\varphi(t)} \quad (8)$$

with small amplitude changes over the considered time interval, i.e., $A(t-\tau)\cong A(t)$ for $0\le\tau\le h$ and $\varphi(t)$ is a continuous and differentiable function of time. The IF of this signal is

$$\omega_i(t)=\frac{d\varphi(t)}{dt}=\varphi'(t) \quad (9)$$

with the phase in the local auto-correlation function of the form

$$3\varphi(t)+\varphi(t-\tau)-4\varphi\left(t-\frac{\tau}{2}\right)=\varphi'(t)\tau-\frac{1}{12}\varphi'''(t)\tau^3+\cdots$$
$$=\varphi'(t)\tau+\Delta\varphi(t,\tau). \quad (10)$$

The IF estimation is based on

$$D(t,\omega)\cong A^8(t)W(\omega-\varphi'(t))*_\omega FT\{e^{-j(1/12)\varphi'''(t)\tau^3+\cdots}\}, \quad (11)$$

where $W(\omega)$ is the FT of $w(\tau)$. Maximal value of $|D(t,\omega)|^2$ is reached at $\omega=\varphi'(t)$ with a possible bias caused by the third and higher order derivatives of phase. Note that in the WD case the maximal value would be reached at $\omega_{WD}=\varphi'(t-h/2)$.

This distribution has been defined with the aim to be real-time instantaneous frequency estimator, in the first place. It also satisfies some other time-frequency representation properties. It preserves shift in time and frequency. For $y(t)=x(t-t_0)\exp(j\omega_o t)$, $D_y(t,\Omega)=D_x(t-t_0,\ \omega-\omega_0)$. In addition for a signal $y(t)=x(t)e^{jat^2/2}$, $D_y(t,\omega)=D_x(t,\omega-at)$. Time marginal is $\int_{-\infty}^{\infty}D(t,\omega)\,d\omega/2\pi=\ |x(t)|^8$. For a scaled version of the signal $y(t)=\sqrt{|a|}x(at)$, $a\neq 0$, this distribution reads $D_y(t,\omega)=D_x(at,\omega/a)$. The time constraint is satisfied, as well, since $D(t,\omega)=0$, when $x(t)=0$.

3. Estimator performance

A discrete-time form definition of (6), at an instant $t=nT$, reads

$$D(t,\omega)=\sum_{k=0}^{N-1}w_h(kT)x(t-2kT)x^{*4}(t-kT)e^{-j2\omega kT}. \quad (12)$$

The lag independent part $x^3(t)$ is omitted, since it will not influence the IF analysis. Consider a noisy signal

$$x(nT)=s(nT)+\varepsilon(nT)=Ae^{j\varphi(nT)}+\varepsilon(nT), \quad (13)$$

where $\varepsilon(nT)$ is a complex i.i.d. white Gaussian stationary noise with zero mean and variance σ_ε^2, and $\varphi(nT)$ are discrete time samples of the differentiable continuous function $\varphi(t)$, with bounded derivatives.

The IF, at a time instant $t=nT$, is estimated by

$$\hat{\omega}(t)=\arg\max_\omega|D(t,\omega)|^2=\arg\max_\omega F(t,\omega), \quad (14)$$

where $F(t,\omega)=D(t,\omega)D^*(t,\omega)$.

In order to analyze the estimators performance, we will linearize $\partial F(t,\omega)/\partial\omega$ around the stationary point where $\partial F(t,\omega)/\partial\omega_{|0}=0$, with respect to the estimation