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A 2-D robust high-resolution frequency estimation approach ☆

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Abstract

This paper deals with a 2-D high-resolution frequency estimation method dedicated to images corrupted by outliers. Outliers are considered here as particular data points that do not fit an assumed model. We propose an efficient subspace estimation algorithm for 2-D complex sinusoids. In this framework the well-known model using the sum of complex exponentials fails for a small fraction of the data set causing the classical estimators to produce inaccurate results. To alleviate this drawback, we propose a new robust iterative Levenberg–Marquardt (LM) approach based method. The proposed approach operates in three main steps. First, we define a weight function based on the influence function which allows the "wrong" data to be detected and corrected. The influence function, which is inspired from the so-called M-estimator, measures the influence of a datum on the value of the estimated parameter. Second, a 2-D extension of the large sample approximation of the maximum likelihood (ML) estimator is developed in order to estimate image parameters. Third, the Levenberg–Marquardt (LM) technique is used to ensure the convergence of the ML estimator by detecting "wrong" data for each iteration. The effectiveness of the proposed method is illustrated by numerical simulations.

Keywords: 2-D exponential signal model; High-resolution spectral analysis; Weighted subspace fitting; M-estimation; Robust estimation

1. Introduction

The frequency content estimation of a sum of two-dimensional complex exponentials from a finite subset of noisy data is a common problem in image processing such as array, radar, seismic, and biomedical applications. To solve the above problem, several approaches have been proposed in the Refs. [1–4]. All the methods exploit a low rank signal

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Nomenclature

$\mathbf{A}^{\mathrm{T}}$	transpose of $\mathbf{A}$
$\mathbf{A}^{\mathrm{H}}$	complex conjugate and transpose of $\mathbf{A}$
$\mathrm{Re}\{\mathbf{A}\}$	real part of $\mathbf{A}$
$\mathbf{A}^{\#}$	pseudo-inverse of $\mathbf{A}$
$\mathrm{Tr}\{\mathbf{A}\}$	trace of $\mathbf{A}$
$\mathbf{A}\otimes\mathbf{B}$	Kronecker product between the matrices $\mathbf{A}$ and $\mathbf{B}$
$\mathbf{A}\odot\mathbf{B}$	Hadamard product between the matrices $\mathbf{A}$ and $\mathbf{B}$
$\delta(x)$	Dirac
$\mathbf{I}_M$	identity matrix of order M
$\mathbf{0}_M$	$M\times M$ matrix of zeros
$\mathbf{0}_{M,N}$	$M\times N$ matrix of zeros
$\mathrm{diag}\{v\}$	diagonal square matrix of size the length of v and its main diagonal consists of the elements of v
$\mathrm{diag}\{\mathbf{V}\}$	vector consisting of the main diagonal of the matrix $\mathbf{V}$
arg(.)	argument of the specified function

subspace version of the autocorrelation matrix decomposition due to time-series features according to the linear combination of a finite number of sinusoidal signals. Avoiding the costly optimization or polynomial root finding for multidimensional harmonic retrieval, these methods provide very efficient estimation techniques increasing the frequency resolution of the classical periodogram.

Considerable literature is devoted to the M-estimation theory [5–9]. This interest is certainly due to the fact that a M-estimator is directly linked to the Maximum likelihood estimator. In contrast, the proposed M-estimator in one dimensional signal processing area is quite limited [10–12]. The approach proposed by Katkovnik [10] addresses the robust M-estimate in the framework of spectral analysis of time-invariant frequency. In his paper, the author develops a robust periodogram without low-rank subspace reduction. In [11,12], polynomial root finding based on Huber's function are proposed and the multidimensional extension is de facto not realistic. In [13], we have recently proposed a 2-D robust approach based on Tukey's function in order to restore the signal subspace.

In this paper, we propose a 2-D robust large sample approximation of the maximum likelihood estimator dedicated to the robust high-resolution spectral method. It is achieved via three main steps. First, we define a weight function based on the influence function [14], which allows outliers in the data to be detected and corrected. The influence function, which is inspired from the so-called M-estimators [15–17], characterizes the bias that a particular error is likely to induce on the solution. Second, we develop a 2-D extension of the large sample ML, namely the 2-D robust weighted subspace fitting (2-D RWSF) method, originally presented in [18] in its 1-D form and only dedicated to Gaussian distribution of the source noise. Third, based on the Levenberg–Marquardt (LM) technique we improve the performances of the 2-D parameter estimation of the 2-D ML estimator by detecting and correcting "wrong" data for each iteration. According to the WSF method, we use the QR-decomposition [19] to perform the pseudo-inversion operation needed in the LM algorithm in order to reduce the computation cost. The method is tested on various examples and the results obtained demonstrate its robustness against outliers.

The remainder of the paper is organized as follows. In Section 2 we present the data model and outliers followed by a short review of the 2-D ESPRIT approach. In Section 3, we describe the new 2-D robust approach for frequency estimation. Section 4 provides simulation results. Section 5 concludes the paper. Finally, the matrix tools used to minimize the cost function are grouped in Appendix A.

2. Problem formulation

2.1. Data model

Let $x(m,n)$ be a scalar observed pixel modelled as a sum of p 2-D cisoids corrupted by noise

$$x(m,n)=\sum_{k=1}^{p}a_k\mathrm{e}^{\mathrm{j}2\pi(f_{1k}m+f_{2k}n)}+b(m,n), \tag{1}$$