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On the efficiency of a bearings-only instrumental variable estimator for target motion analysis

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Abstract

The maximum-likelihood (ML) estimator for bearings-only target motion analysis does not admit a closed-from solution and must be implemented iteratively. Iterative ML estimators require an initialization close to the true solution to avoid divergence. Recently a closed-form asymptotically unbiased instrumental variable estimator has been proposed to alleviate the convergence problems associated with iterative ML estimators. This paper establishes the asymptotic efficiency of the closed-form instrumental variable estimator by showing that its error covariance matrix approaches the Cramer–Rao lower bound for sufficiently small bearing noise as the number of measurements tends to infinity.

Keywords: Target motion analysis; Bearings-only target tracking; Asymptotic efficiency; Maximum likelihood; Pseudolinear estimator; Bias compensation; Instrumental variables

1. Introduction

The objective of bearings-only target motion analysis is to estimate the position, velocity and acceleration of a target from noisy bearing angle measurements collected by a moving observer. The moving observer, which is also known as the ownship, can be an aircraft, a ship or an unmanned aerial vehicle (UAV). Target motion analysis or target tracking is an important practical problem with applications in radar, sonar and mobile communications, to name but a few.

Bearings-only target tracking has been an active area of research for several decades. Given the nonlinear relationship between the bearing measurements and the target location, the target tracking problem can be solved by the extended Kalman filter (EKF) [1]. The EKF algorithm is known to exhibit divergence problems in Cartesian coordinates [1], and a remedy has been found by implementing the EKF in modified polar coordinates [2]. Because of the recursive nature of the EKF algorithms, good initialization is a must in order to avoid divergence [14]. The

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Nomenclature

$\boldsymbol{a}$	constant target acceleration
$\boldsymbol{b}$	"observation" vector of the pseudolinear estimator
d_k	target range at time t_k
$\hat{d}_k$	bias-compensated pseudolinear estimate of d_k
$\boldsymbol{F}$	"data" matrix of the pseudolinear estimator
$\boldsymbol{F}_o$	noise-free $\boldsymbol{F}$
$\boldsymbol{G}$	instrumental variable matrix
$\boldsymbol{J}(i)$	Jacobian matrix of estimated bearing error
$\boldsymbol{K}$	diagonal covariance matrix of bearing noise
$\boldsymbol{M}_k$	matrix that transforms target motion parameters to target location at time t_k
n_k	bearing noise
$\boldsymbol{v}_k$	unit vector orthogonal to true bearing line at time t_k
N	the number of bearing measurements collected over the period $[0, T)$
$\boldsymbol{\eta}$	noise vector obtained from nonlinear functions of the bearing noise
$\boldsymbol{p}_0$	initial target location
$\boldsymbol{p}_k$	target location vector at time t_k
$\boldsymbol{r}_k$	observer location vector at time t_k
σ_n^2	bearing noise variance
t_k	bearing measurement time instant
T	bearing observation time interval
θ_k	bearing angle at time t_k
$\tilde{\theta}_k$	bearing measurement at time t_k
$\boldsymbol{u}_k$	unit vector orthogonal to measured bearing line at time t_k
$\boldsymbol{v}_0$	initial target velocity
$\boldsymbol{W}$	estimated weighting matrix for $\hat{\boldsymbol{\xi}}_{\mathrm{WIV}}$
$\boldsymbol{W}_o$	true weighting matrix for $\hat{\boldsymbol{\xi}}_{\mathrm{WIV}}$
$\boldsymbol{\xi}$	target motion parameter vector
$\hat{\boldsymbol{\xi}}_{\mathrm{BCLS}}$	bias-compensated pseudolinear estimate of $\boldsymbol{\xi}$
$\hat{\boldsymbol{\xi}}_{\mathrm{LS}}$	pseudolinear estimate of $\boldsymbol{\xi}$
$\hat{\boldsymbol{\xi}}_{\mathrm{MIV}}$	iterative modified instrumental variable estimate of $\boldsymbol{\xi}$
$\hat{\boldsymbol{\xi}}_{\mathrm{ML}}$	maximum-likelihood estimate of $\boldsymbol{\xi}$
$\hat{\boldsymbol{\xi}}_{\mathrm{WIV}}$	closed-form weighted instrumental variable estimate of $\boldsymbol{\xi}$

maximum-likelihood (ML) estimator for target tracking [24,20] does not admit a closed-form solution and is therefore implemented as an iterative search algorithm. Iterative ML estimators are not only computationally expensive, but they also exhibit convergence difficulties unless initialized close to the solution. A least-squares (LS) estimator with closed-form solution, which is referred to as the pseudolinear estimator, was developed in [15]. Despite its simplicity and low computational complexity, the pseudolinear estimator suffers from severe bias [3,20].

To overcome the bias of the pseudolinear estimator, two iterative approaches have been developed, viz., the modified instrumental variable (MIV) estimation algorithm [20] and the recursive instrumental variable (IV) estimator [4]. Neither of these IV algorithms have a closed-form solution since they rely on an iterative process to compute an instrumental variable matrix. They also require appropriate initialization in order to avoid divergence. The convergence properties of iterative bearings-only tracking algorithms are analyzed in detail in [13].

Several unbiased closed-form solutions have been proposed. Some of the key algorithms in this category are the reduced-bias closed-form tracker [11], the efficient unbiased estimator [23], and the closed-form reduced-bias pseudolinear estimator [19]. The first algorithm requires large range-to-baseline ratio and small bearing noise for unbiasedness, and the last two algorithms assume a multi-leg constant-velocity trajectory for the observer.

Recently, we have proposed a new asymptotically unbiased closed-form tracking algorithm based on instrumental variables and a bias-compensated pseudolinear estimator [6]. This algorithm considers a more general target motion model than the previous algorithms, which permits the target to have a constant acceleration. It also makes no restrictive assumptions about the