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Constructive role of noise in signal detection from parallel arrays of quantizers

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Abstract

A noisy input signal is observed by means of a parallel array of one-bit threshold quantizers, in which all the quantizer outputs are added to produce the array output. This parsimonious signal representation is used to implement an optimal detection from the output of the array. Such conditions can be relevant for fast real-time processing in large-scale sensor networks. We demonstrate that, even for suprathreshold input signals, the presence of independent noises added to the thresholds in the array, can lead to a better performance in the optimal detection. We relate these results to the phenomenon of suprathreshold stochastic resonance, by which nonlinear transmission or processing of signals with arbitrary amplitude can be improved by added noises in arrays.

Keywords: Detection; Quantizer; Nonlinear arrays; Sensor arrays; Suprathreshold stochastic resonance

1. Introduction

When signal and noise are coupled nonlinearly, there exists a possibility for the noise to interact constructively with the signal, so that the presence of the noise can reveal beneficial. Stochastic resonance (SR) describes this possibility of a constructive action of the noise. Introduced some 20 years ago in the context of nonlinear physics, SR has progressively been reported in many areas, under many different forms, with various types of signals, nonlinear processes and measures of performance receiving improvement from the noise (see [1–5] for recent surveys). SR has also been applied specifically to standard signal processing problems, for instance to detection [6–9] or estimation [10].

So far, most studies have shown SR with nonlinear systems presenting thresholds or potential barriers, and in which the noise brings assistance to a small subthreshold signal in overcoming the nonlinearity for a more efficient response. Recently, another mechanism of improvement by noise was introduced under the

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name of suprathreshold SR [11]. This SR applies to threshold devices driven by a signal already above threshold which needs no assistance to overcome it. SR is thus absent in a single device, but appears when the devices are associated in a parallel array. Independent noises injected on the devices have the ability to increase the efficacy of representation of the signal by the array compared to a single device with no added noise. This translates into the possibility of improving various measures of performance, depending on the task at hand, by addition of noises in the array. This novel suprathreshold SR has been observed with the mutual information [11–13] and the input–output correlation [14] in random signal transmission, with a signal-to-noise ratio in periodic signal transmission [15], with the Fisher information in signal estimation [16].

In the present paper, we consider the same type of parallel arrays of comparators as used in previous suprathreshold SR studies [11–16], and we investigate them in the framework of an optimal detection task. We emphasize, as also done in the previous studies, that such arrays of nonlinearities bear similarities and significance to several areas including known technologies (like sonar arrays [17], flash analog-to-digital converters [18]), or promising avenues (like neural processing [19], cochlear implants [20], artificial vision [21], or other new-generation sensing devices [22]). The possibility of noise enhancement of information processing in nonlinear arrays, with its various modalities, is thus a new property with rich potentialities to be explored, an endeavor to which the present study participates.

2. Optimal detection from the output of a nonlinear parallel array

We consider a detection task where an input signal $s(t)$ can be one of two known signals, $s(t) \equiv s_0(t)$ with prior probability P_0, or $s(t) \equiv s_1(t)$ with prior probability $P_1 = 1 - P_0$. The input signal $s(t)$ is buried in an input noise $\xi(t)$ with probability density function $f_\xi(u)$. This yields the input signal–noise mixture $s(t) + \xi(t) = x(t)$. This mixture $x(t)$ is observed by means of a parallel array of N threshold comparators or one-bit quantizers, following the setting of [11–13]. We arrange for the possibility of a noise $\eta_i(t)$, independent of $x(t)$, to be added to $x(t)$ before quantization by quantizer i. Quantizer i, with threshold θ_i, delivers the output

$$y_i(t) = U[x(t) + \eta_i(t) - \theta_i], \quad i = 1, 2, \dots N, \tag{1}$$

where $U(u)$ is the Heaviside function, i.e. $U(u) = 1$ if $u > 0$ and is zero otherwise. The response $Y(t)$ of the array is obtained by summing the outputs of all the quantizers, as

$$Y(t) = \sum_{i=1}^{N} y_i(t). \tag{2}$$

The array output $Y(t)$ of Eq. (2) is measured at M distinct times t_k, for $k = 1$ to M, so as to provide M data points $Y_k = Y(t_k)$. Each one of the Y_k can assume only $N + 1$ discrete integer values from 0 to N, so a total of $(N + 1)^M$ discrete states are accessible to the data $\boldsymbol{Y} = (Y_1, \dots, Y_M)$. We then want to use the data $\boldsymbol{Y}$ to decide whether the noisy input $x(t)$ is formed by $\xi(t)$ mixed to $s_0(t)$ (hypothesis H_0) or to $s_1(t)$ (hypothesis H_1).

According to classical detection theory [23], the detector that minimizes the overall probability of detection error P_{er}, uses the likelihood ratio $L(\boldsymbol{Y})$ to implement the test

$$L(\boldsymbol{Y}) = \frac{\Pr\{\boldsymbol{Y}|H_1\}}{\Pr\{\boldsymbol{Y}|H_0\}} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{P_0}{P_1}, \tag{3}$$

and in doing so achieves the minimal P_{er} expressable as

$$P_{er} = \frac{1}{2} - \frac{1}{2} \sum_{\boldsymbol{Y}} |P_1 \Pr\{\boldsymbol{Y}|H_1\} - P_0 \Pr\{\boldsymbol{Y}|H_0\}|, \tag{4}$$

where the sum in Eq. (4) runs over the $(N + 1)^M$ states accessible to the data $\boldsymbol{Y}$.

We will consider here that the N threshold noises $\eta_i(t)$ are white (strict sense), mutually independent, and identically distributed with cumulative distribution function $F_\eta(u)$ and probability density function $f_\eta(u) = \mathrm{d}F_\eta/\mathrm{d}u$. We also consider that the input noise $\xi(t)$ is white, just as the threshold noises $\eta_i(t)$ are. The conditional