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Information-theoretic assessment of multi-dimensional signals

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Abstract

This work focuses on estimating the information conveyed to a user by multi-dimensional digitised signals. The goal is establishing the extent to which an increase in radiometric resolution, or equivalently in signal-to-noise ratio (SNR), can increase the amount of information available to users. Lossless data compression is exploited to measure the "useful" information content of the data. In fact, the bit-rate achieved by the reversible compression process takes into account both the contribution of the "observation" noise, i.e. information regarded as statistical uncertainty, whose relevance is null to a user, and the intrinsic information of hypothetically *noise-free* samples. Once the parametric model of the noise, assumed to be possibly non-Gaussian, has been preliminarily estimated, the mutual information between noise-free signal and recorded noisy signal is easily estimated. However, it is desirable to know what is the amount of information that the digitised samples would convey if they were ideally recorded without observation noise. Therefore, an entropy model of the source is defined and such a model is inverted to yield an estimate of the information content of the noise-free source from the code rate and the noise model. Results are reported and discussed both on a simulated noisy image and on true hyperspectral data (220 spectral bands) recorded by the AVIRIS imaging spectrometer.

Keywords: Airborne Visible/InfraRed Imaging Spectrometer (AVIRIS); Correlation analysis; Generalised Gaussian PDF; Hyperspectral imagery; Information-theoretic assessment; Lossless compression; Multivariate regression; Noise estimation; Noise modelling; Parametric entropy modelling

1. Introduction

The term *raster data* denotes digitised samples of signals produced by sensors capable to detect and measure a physical property defined in a multi-dimensional domain. Examples of such signals are audio signals (1D), scanned

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monochrome images (2D), video signals (3D), and generally most of remote-sensing data, like those produced by multi-spectral scanners and imaging spectrometers. A common feature of all raster data is the presence of noise, auto-correlation along each of their dimensions and a generally large number of digitisation levels: at least 256, but also up to 65,536; typically between 1024 and 4096.

This work focuses on defining a procedure suitable for quantifying the information content of digitised multi-dimensional signals, more specifically mono- and multi-band images. Information-theoretic assessment of imaging systems is presently an open problem [12,19,2]. Accurate estimates of the *entropy* of an image source can only be obtained provided that the data are uncorrelated. Hence, data decorrelation must be considered in order to suppress or largely reduce the correlation existing in natural images. Indeed, entropy is a measure of statistical information, that is of uncertainty of symbols emitted by a source. Hence, any observation noise introduced by the imaging sensor will result in an increment in entropy, that is accompanied by a decrement of the information content useful in application contexts, according to Shannon's information theory [22].

Modelling and estimation of the noise must be preliminarily carried out [3] in order to quantify it contribution to the entropy of the *observed* source. Modelling of information sources is important as well, to assess the role played by signal-to-noise ratio (SNR) in determining the extent to which an increase in radiometric resolution can increase the amount of information available to users.

The models that are exploited are simple, yet adequate for describing first-order statistics of memory-less information sources and auto-correlation functions of noise processes, typically encountered in digitised raster data. The mathematical tractability of models is fundamental for deriving an information-theoretic closed-form solution yielding the entropy of the noise-free signal from the entropy of the observed noisy signal and the estimated noise model parameters. Furthermore, the present work was motivated by the availability of extremely advanced lossless compression methods, capable to attain the ultimate compression ratio regardless of any issues of computational complexity [6,7].

The remainder of this paper is organised as follows. The information-theoretic fundamentals underlying the analysis procedure are reviewed in Section 2. Section 3 presents the information-theoretic procedure step by step, starting from the assumed noise model, source decorrelation by DPCM, parametric entropy modelling of memory-less information sources via generalised Gaussian densities. Section 4 reports experimental results on a simulated noisy image (2D signal) and on a test hyperspectral image (3D signal) acquired by the AVIRIS imaging spectrometer. Concluding remarks are drawn in Section 5.

2. Information-theoretic problem statement

If we consider a discrete multi-dimensional signal as an information source S, its average information content is given by the entropy $H(S)$ [9]. An acquisition procedure originates an observed digitised source $\hat{S}$, whose information is the entropy $H(\hat{S})$. $H(\hat{S})$ may not be adequate to measure the amount of acquired information, since the observed source will generally not coincide with the digitised original source, mainly because of the observation noise. Furthermore, the source may be not exactly band-limited by half of the sampling frequency; hence, the non-ideal sampling is responsible for an additional amount of noise generated by aliasing. Therefore, only a fraction of the original source information is conveyed by the digitised noisy signal.

The amount of source information that is not contained in the digitised samples is measured by the conditional entropy $H(S|\hat{S})$, or equivocation, that is the residual uncertainty on the original source when the observed source is known. The contribution of the overall noise (i.e. *aliasing* and acquisition) to the entropy of the digitised source is measured by the conditional entropy $H(\hat{S}|S)$, which represents the uncertainty on the observed source $\hat{S}$ when the original source S is known. Therefore, the larger the acquisition noise, the larger $H(\hat{S})$, even if the amount of information of the original source that is available from the