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Short Communication

Application of symmetric fuzzy linear programming in life cycle assessment

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Abstract

Life cycle assessment (LCA) is known to entail multiple objective decision-making in the analysis of tradeoffs between different environmental impacts. The work of Azapagic and Clift in the late 1990s illustrates the use of multiple objective linear programming (MOLP) in the context of LCA. However, it will be noted that their approach yields a range of Pareto optimal alternatives from which the decision-maker must ultimately select the final solution. An alternative approach making use of Zimmermann's symmetric fuzzy linear programming method is proposed and illustrated with a simple case study. The procedure effectively yields a single, optimal compromise solution.

Keywords: Life cycle assessment; Fuzzy linear programming

Software availability

SFLP template for up to 20 alternatives and 20 criteria is available as a Microsoft Excel spreadsheet file. The model can be solved using the standard Solver add-in.

1. Introduction

Life cycle assessment (LCA) is a methodology for studying the environmental flows and effects of a technological system (i.e., product, process or service) on a cradle-to-grave basis. LCA considers both direct and indirect impacts to give a more accurate picture of how the system affects the environment. The impacts are normally quantified for a given reference volume of final output known as the *functional unit*. Furthermore, impacts may occur through different pathways such as acid rain formation, global warming or natural resource depletion (ISO, 1997; Azapagic, 1999). Standard LCA consists of the following components (ISO, 1997):

- Goal and scope definition
- Inventory analysis
- Impact assessment – consisting of classification, characterization and valuation
- Interpretation

Impact assessment involves quantifying environmental impacts generated by different environmental flows and pollutants. Since there are different pathways by which the environment can be affected, analysis of a system will usually involve multiple objectives or criteria. Thus in comparing different options it becomes necessary to analyze tradeoffs between potentially conflicting goals. Azapagic and Clift (1995, 1999a,b) employed multiple objective linear programming (MOLP) techniques for LCA. Their approach generates

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a range of Pareto-optimal or non-dominated solutions to a given LCA problem. However, selection of the final solution to be used is left to the decision-maker.

3. Fuzzy LP model for LCA

Fuzzy linear programming (FLP) techniques offer computationally efficient alternatives to stochastic programming for optimization under uncertainty (Lai and Hwang, 1992; Rommelfanger, 1996; Sadiq and Husain, 2005). Zimmermann (1992) developed a symmetric FLP (SFLP) formulation where constraints are made flexible by introducing the concept of degree of feasibility. In the SFLP model, multiple objective functions can thus be treated as fuzzy constraints so that the global objective is to maximize the degree of feasibility of all the fuzzy constraints (or objectives) simultaneously. The key features of SFLP are:

- Crisp or non-fuzzy constraints are converted to fuzzy constraints by introducing tolerances. This modification introduces the concept of degree of satisfaction of a constraint, *a*, bounded in the interval [0, 1].
- An aspiration level is identified for each objective function, such that optimization entails maximizing the degree to which the objectives are satisfied. This step involves identifying the best and worst values for each objective. The degree to which an objective is satisfied is also bounded in the interval [0, 1].
- Objectives and constraints are treated in the same manner in SFLP – hence the use of the term *symmetric*. A new variable, α, is introduced in the model which serves to simultaneously modulate the degrees of satisfaction of all the constraints or objectives. The SFLP is then formulated to maximize α, which in effect is the global degree of satisfaction in the model.

Use of SFLP results in a very compact model, requiring only one additional variable and objective function (α) compared to an ordinary LP. Another key advantage is that the model remains linear and can be solved using the well-known simplex algorithm for LPs. Recent environmental applications of SFLP have been demonstrated, including water reuse network synthesis (Tan and Cruz, 2004) and data reconciliation for LCA (Tan and Culaba, 2004).

For a product-mix problem where *m* different competing alternatives are evaluated based on *n* different environmental criteria, a typical objective might be to determine the fractional or percentage contribution of each alternative to the total system output. This problem assumes the alternatives serve the same purpose. An example of such a problem is the generation of electricity from different energy sources. Grid power available to the public consists of a mix of electricity produced in different ways. To determine the optimal product mix based on multiple environmental criteria, the following SFLP model can be used:

$$\max \alpha \tag{1}$$

Subject to:

$$\sum_i a_{ij} x_i \le b_j (1-\alpha) \quad \forall j \tag{2}$$

$$\sum_i x_i = 1 \tag{3}$$

$$\alpha \le 1 \tag{4}$$

$$\alpha, x_i \ge 0 \quad \forall i \tag{5}$$

Variables

α global degree of feasibility
x_i fractional share of (*i*) in total product mix

Parameters

a_{ij} environmental impact in category (j) per unit of product (i)
b_j maximum tolerable environmental impact in category (j) per unit combined output

The overall objective is to maximize overall degree of feasibility (Eq. 1). Fuzzy constraints for environmental impact (Eq. 2) are structured such that when $\alpha = 1$, the right hand side becomes zero, and when $\alpha = 0$, the right hand side becomes b_j. A final crip (non-fuzzy) constraint ensures that the contributions of the alternative sum up to unity (Eq. 3). The concept of a fuzzy constraint is illustrated in Fig. 1. There is a gradual transition from feasibility to infeasibility. For simplicity this transition is described by a linear membership function, which allows the linearity of the model to be preserved as well.

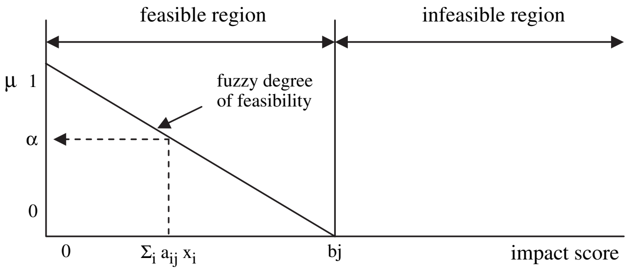

Fig. 1. Fuzzy constraint and degree of feasibility.