Journal of Colloid and Interface Science 364 (2011) 505–511

Contents lists available at SciVerse ScienceDirect

# Journal of Colloid and Interface Science

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# Effect of disjoining pressure on terminal velocity of a bubble sliding along an inclined wall

Lorena A. Del Castillo [a], Satomi Ohnishi [a,*], Lee R. White [b], Steven L. Carnie [c], Roger G. Horn [d]

[a] Ian Wark Research Institute, University of South Australia, Mawson Lakes, SA 5095, Australia
[b] School of Mathematics and Statistics, University of South Australia, Mawson Lakes, SA 5095, Australia
[c] Department of Mathematics and Statistics, The University of Melbourne, Victoria 3010, Australia
[d] Institute of Technology Research and Innovation, Deakin University, 221 Burwood Highway, Victoria 3125, Australia

ARTICLE INFO

*Article history:*
Received 14 January 2011
Accepted 19 August 2011
Available online 26 August 2011

*Keywords:*
Sliding bubble
Terminal velocities
Disjoining pressure
Hydrodynamic forces
Double-layer forces
Electrolyte concentration

ABSTRACT

The influence of salt concentration on the terminal velocities of gravity-driven single bubbles sliding along an inclined glass wall has been investigated, in an effort to establish whether surface forces acting between the wall and the bubble influence the latter's mobility. A simple sliding bubble apparatus was employed to measure the terminal velocities of air bubbles with radii ranging from 0.3 to 1.5 mm sliding along the interior wall of an inclined Pyrex glass cylinder with inclination angles between 0.6 and 40.1°. Experiments were performed in pure water, 10 mM and 100 mM KCl solutions. We compared our experimental results with a theory by Hodges et al. [1] which considers hydrodynamic forces only, and with a theory developed by two of us [2] which considers surface forces to play a significant role. Our experimental results demonstrate that the terminal velocity of the bubble not only varies with the angle of inclination and the bubble size but also with the salt concentration, particularly at low inclination angles of ~1–5°, indicating that double-layer forces between the bubble and the wall influence the sliding behavior. This is the first demonstration that terminal velocities of sliding bubbles are affected by disjoining pressure.



## 1. Introduction

Many industrial processes such as mineral flotation, water treatment, and emulsification, are carried out under conditions of multiphase flow, in which bubbles play a key role in determining the process efficiency. Hence, the motion of bubbles in different configurations has been studied extensively in order to understand the kinetics of bubbles interacting with solid surfaces. A large number of studies has been conducted on vertically rising bubbles in unbounded or confined geometries [3–22] which have provided insights on air–liquid interfaces of free rising bubbles and bubbles influenced by solid surfaces. Since a lot of processes in reality involve not only vertical movement of bubbles but also bubble motion along angled surfaces in liquid media, studies of bubbles moving parallel to inclined surfaces [23–41] are also of significant interest in different fields such as engineering, medicine and various industries including petroleum and food.

To understand the kinetics of bubbles sliding along solid surfaces, it is essential to identify the relevant forces acting on the bubble. The only pertinent theory we found in the literature is that of Hodges et al. [1] which considers gravitational and hydrodynamic forces in predicting the behavior of drops sedimenting down a gently inclined plane. The theory determines the thickness $h_o$, of the thin liquid film separating the drop (or bubble) and the solid wall, from a normal force balance of buoyancy against hydrodynamic pressure forces. Hodges et al. compared the velocities predicted by their theory with an experimental study on air bubbles sliding along inclined planes in viscous liquid (silicone oil) performed by Aussillous and Quéré [31]. The velocities predicted by the theory of Hodges et al. were greater than the experimental measurements of Aussillous and Quéré, indicating that the theory has not captured all of the physics in this system.

Another type of force that could be affecting the sliding velocity, albeit indirectly, is a surface force acting between the bubble and the wall, such as a van der Waals force or an electrical double-layer repulsion. The effect of such forces, expressed as an additional "disjoining" pressure in the thin layer of liquid between the bubble and wall, would be to deform the bubble, thereby changing the hydrodynamic forces acting on it. Of course hydrodynamic forces can also deform the bubble, as discussed by Hodges et al., making a theory that incorporates both disjoining and hydrodynamic pressures quite complex.

A bare glass surface (without modification/capping by chemical reagents) in water (pH 5–7) is negatively charged, since the isoelectric point of glass is observed to be around pH 2 [42,43]. The surface potential of an air–water interface is generally negative [44–51]. Since glass surfaces and air–water interfaces are both

* Corresponding author. Fax: +61 8302 3755.
*E-mail address:* satomi.onishi@unisa.edu.au (S. Ohnishi).

0021-9797/$ - see front matter © 2011 Elsevier Inc. All rights reserved.
doi:10.1016/j.jcis.2011.08.046

negatively charged in water, one could expect some influence from electrical double layer interaction between them. Thus we think that disjoining pressure could play a role in the dynamics of a bubble sliding along inclined glass surfaces in aqueous liquid. To investigate this, we initiated a two-pronged attack based on experiments and theoretical development.

The theory assumes, as its starting point, that the bubble is effectively flattened by its DLVO interaction with the wall, so that the thickness $h_o$ of a uniform thin film separating the bubble (approximated by a truncated sphere) is determined by the surface charges on the wall and the bubble, and the electrolyte concentration in the aqueous phase. The terminal velocity $U_T$ of a bubble sliding along an inclined surface is found to depend on the radius $A$ of the bubble, the angle of inclination $\phi$ of the surface, and $h_o$ [2].

In parallel with the theoretical development, we also explored experimentally whether disjoining pressure has an effect on the terminal velocity of the sliding bubble in aqueous solutions. To achieve this we employed a simple sliding bubble apparatus to measure the terminal velocity of a bubble sliding under the force of gravity along an inclined surface in an aqueous medium, with various concentrations of electrolyte because this is well known to influence double-layer forces.

The body of literature on bubbles sliding along inclined surfaces in liquid mostly reports on effects of bubble size [27,31,33,37,40], bubble shape [25], size of liquid channel [24–26,30], angle of inclination [23–28,30,31,33,36–38,41], surfactants [33,34,40], gas type [40], liquid temperature [26,40,52], and other liquid properties [23,24,26,28,30,31,52] such as viscosity, density, and surface tension on the dynamics of the sliding bubble. However, salt concentrations of liquid media, to our knowledge, have not yet been a parameter of any study in bubbles sliding along inclined surfaces.

In this article we present our experimental observations on the effects of salt concentration together with that of the bubble size and angle of inclination on the terminal velocity of the sliding bubble. We also present a comparison between our experimental results and the two theories discussed above, those due to Hodges et al. [1] and White and Carnie [2].

## 2. Materials and methods

### 2.1. Sliding bubble apparatus

The sliding bubble apparatus employed in this study (Fig. 1) was the same device used in bubble coalescence experiments reported previously [53] to investigate the effects of salt concentration and speed of approach on the coalescence time of a bubble rising to meet the surface of the liquid. The experimental set-up consisted of a sealed graduated glass cylinder (100 ml, Pyrex, England), containing the liquid medium, attached parallel to a tilting platform. The cylindrical geometry ensures that the sliding bubble follows a straight path, while the internal diameter of the cylinder (37 mm) is large enough that curvature of the solid surface is negligible on the mm scale of the bubble. A Teflon cap sealing the cylinder had two holes: one where a long stainless steel needle (Hamilton, 26 s gauge, 38 mm long) is fitted and the other one (0.47 mm) acting as a very small air vent. A bubble, formed by turning the screw-threaded plunger of an airtight glass syringe (1 ml, Gastight #1001, Hamilton, USA), was released at the lower end of the cylinder via the needle and rose until it reached the cylinder wall. As the bubble slid up along the top wall of the cylinder, its size and terminal velocity were measured with the aid of a high speed camera (CMOS Ultra II, 1.3 megapixel with NMV-6 Navitar 6 mm f1.4 lens) attached to the platform. Images of bubbles sliding at the middle part of the cylinder where the bubble has already attained terminal velocity were captured with frame rates and spatial resolutions depending on the angle of inclination: 91 frames per second at a spatial resolution of 768 × 568 pixels for low angles of inclination ($\phi$ = 0.6–4.2°); and 147 frames per second at 500 × 550 pixels for higher angles ($\phi$ = 10–40°). Graduations on the outside of the cylinder were used to calibrate the camera magnification and to identify the position of the bubble as a function of time. The angle of inclination was varied by a screw mechanism, and determined by measuring the height of a laser spot, projected by a laser pointer attached parallel to the platform, at a known distance 1–2 m away from the pivot point. A backlight (white LED backlight NT57-820, Edmund Optics) and two soft fiber optic lights (Fibreoptic Illuminator Model 15001, Fibreoptic Lightguides, Australia) were positioned as shown in the figure to obtain sharp and shadowless bubble images.

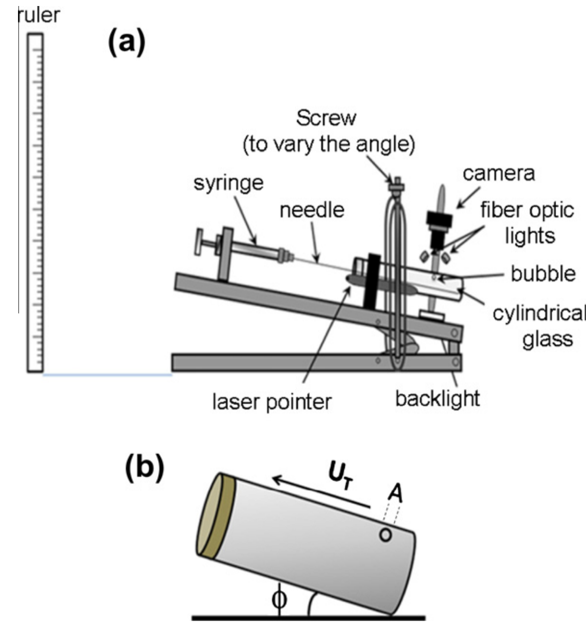


**Fig. 1.** Sliding bubble apparatus employed to generate bubbles sliding at different speeds and to measure their sizes and terminal velocities: (a) schematic illustration of the experiment; (b) an illustration showing the measurable parameters: angle of inclination, $\phi$, bubble terminal velocity $U_T$, and bubble radius, $A$.

ImageJ freeware (Ver. 1.38x, downloaded from http://rsweb.nih.gov/download.html) was used to measure the size and speed of the bubbles. Estimated errors were ±0.02 mm in diameter; a maximum of ±0.1° in inclination angle for $\phi$ < 10° and a maximum of ±0.2° for $\phi$ > 10°. For the speed, estimated errors depend on the speed range: ±0.01 mm/s at very low speeds ($U_T \sim$ 1–3 mm/s); ±0.33 mm/s for $U_T \sim$ 10 mm/s; 7.1 mm/s for $U_T \sim$ 100 mm/s.

We determined $U_T$ by monitoring the velocity of a bubble after it was released from the end of the needle. It was observed that the released bubble initially bounced several times or moved in a nonlinear (zigzag) path for a distance of ~10 mm before sliding parallel to the wall. When the bubble started sliding parallel to the wall the sliding velocity was unstable during the first ~40 mm of travel. For example, for a bubble with radius of 1.2 mm sliding along a wall inclined at $\phi$ = 5°, the speed varied from ~43 mm/s to 77 mm/s over a distance of ~40 mm. The sliding velocity was stabilised (~64 mm/s) after the bubble has travelled ~50 mm. Since all sliding velocities were stabilised at a distance of ~50 mm from the point the bubbles first touched the wall, we took the velocity at a distance of ~55 mm from that point as $U_T$.

### 2.2. Water, salt and cleaning procedures

Water was taken from a high-purity water system (MilliQ, Element) with a resistivity reading (while the water was still in the