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Original article

Quaestiones geometriae in the Amphitheatre of Tarragona

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ABSTRACT

The Roman amphitheatre in Tarragona was built in the first half of the second century AD. We present a study of its formal layout based upon classical discussions of the construction of Roman amphitheatres through determination of the geometrical forms involved in their layout, *ellipsis* and *ovum*. The discussion considers the elevation of conical sections with the ellipse compared to the lowered forms derived from the circle with the oval – figures which are formally often confused. The question considered in this study – a determination of the elevation of the amphitheatre in Tarragona – is based upon a purely geometric analysis; we explicitly avoid considering the instruments required to construct the curve that creates the shape of the amphitheatre. A land survey of the area around the arena enabled us to establish the dimensions of its main axes, the curve described by the original remains of the podium, and the start of the seating tiers. Because the arena apparently has an elliptical layout, our approach to the problem involved determining the four- and eight-centred ovals that provided the best approximation for the only ellipse that fits into the axes that we determined, and we derive statistical confirmation from the deviations of the various figures from the physical reality of the building.

1. Introduction

The city of Tarraco, known nowadays as Tarragona (Spain), was capital of the Hispania Citerior, and one of the few cities during Flávia dynasty, which built an amphitheatre next to the Mediterranean Sea. The Roman amphitheatre in Tarragona was built in the first half of the second century AD. The amphitheatre is situated in an area that was previously used for funeral rites, as bounded by the beach known as Playa del Milagro and the section of the Via Augusta, which reached the eastern side of the city walls.

The amphitheatre is part of the Monumental Set. It is a consequence of the influence of multiple cultures. A Visigoth church was built in the middle of the VI century. Over this church, a Christian church was built in 1154. Later, a Trinity abbey was installed, and from 1576 to 1780, a Diocesan Hostelry (1780) and penal (1792–1908) institution were located on the site. The Spanish State conceded the property to the Tarragona Common Council in 1910. Since 1924, the Amphitheatre has been designated a *Monumento Arquitectónico-Artístico*.

The layout of the amphitheatre has raised various geometrical hypotheses. These hypotheses include an ellipse, such as in the Coliseum of Rome [1–4] and in the amphitheatre of Pompeii [5], a four-centred oval, such as are observed in the Coliseum [6], in Arles and Thysdrus in El-Jem, [7] and in Durazzo [8]. There are even hypothesis of layouts with eight-centred ovals for the Coliseum [9–11]. Ejnar Dyggve's approach in Thessaloniki uses a 12-centred oval [12].

Various approaches have been used to assess the figures of the ellipse and the oval. In the field of architecture, many of these approaches are based upon the constructions of Sebastiano Serlio (1475–1554) [13], completed from the mathematical dimension of both figures [14,15]. These approaches have also been considered from the point of view of the construction and the mechanics of ovoid dome structures [16], the techniques of sternotomy [17], and other fields, such as optics and the principles of geometric perspective [18].

The objectives of this study are to map the layout of the Tarragona amphitheatre and to determine if its form is an oval or an ellipse. The determination of the formal geometry is based on a topographical survey of the building. Surveys with manual, tachometric and laser-scan tools use some of the leading methodologies available, such as those used in the Coliseum [19,20], the Verona Arena [21] and in the London Guildhall [22].

2. Oval and ellipse philosophy

Our approach to oval and ellipse layout is studied from references with direct contact with Roman and late-Roman

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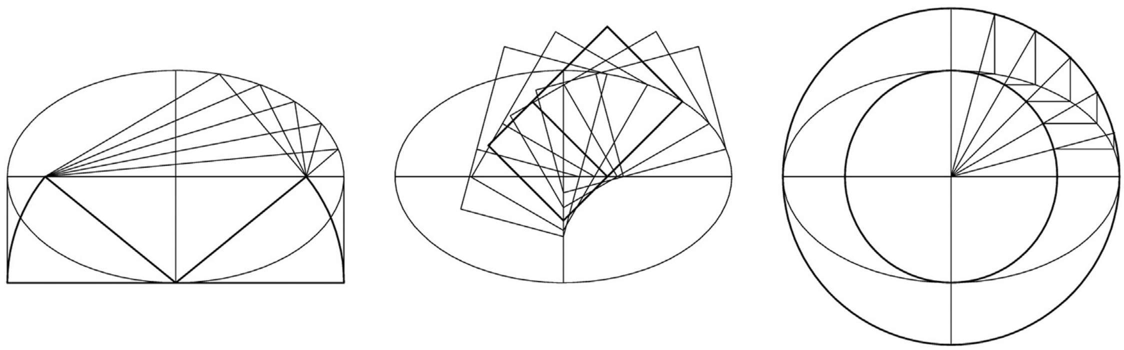

Fig. 1. Systems for drawing the ellipse.

buildings. The first study of the ellipse is attributed to Menaechmus (c.380–c.320 BC) [23]. The ellipse was determined to be the cross-section of a cone by Archimedes of Syracuse (c.287–212 BC) in *Conoids and Spheroids* (IV–VI) [24] and by Apollonius of Perga (c.247–205 BC) at the beginning of Book I (VII–IX) of the *Conics* [25]. This is in addition to the oblique section of a cylinder with a circular directrix proposed by Pappus of Alexandria (c.290–c.350), in Book VIII of the *Collection*, (pp. 13–17) [26].

From the practical standpoint, the oval was used in Egyptian architecture as a practical approach to the ellipse. Vaults were built in Egyptian architecture, *anse de panier surhaussée*, and laid out using three centres [27]. The Euclidian Optics, Prop. XXXVI, considers the appearance of a circular wheel, with an oblong shape, based on two different axes [28]. The Gromatici appear to describe an oval, *ex pluribus circulis forma sine angulo*, using several centres of circumference, in the *Expositio et ratio omnium formarum* (V.9) attributed to Balbus Mensor (fl. c.100) [29]. In Book I.1 *De aedificis* (561), Procopius of Caesarea (c. 490–565) emphasizes knowledge of the ellipse of Anthemius of Tralles's (c. 474–c.558) reflective properties. Anthemius of Tralles built the Hagia Sophia, and, in his treatise on optical mathematics, ΠΕΡΙ ΠΑΡΑΔΟΞΩΝ ΜΜΧΑΝΗΜΑΤΩ, he considers the tangent of the ellipse using its foci and its bisector [30]. The construction of an ellipse using a rope or a chain is thus attributed to him [31].

Sebastiano Serlio (1475–1554) drew and studied Roman buildings. Serlio's *Il Primo libro d'architettura, Le premier libre d'Architecture* (1545) had the layout of an oval. Serlio identified, in Book I, four ways to lay out an oval:

(i) with generatrices forming equilateral triangles;
(ii) three circles;
(iii) two perfect squares;
(iv) two circles [32,33].

With the constructs (i), (ii) and (iv), one axis is parameterized and the other is deduced. In (iii), it is impossible to establish the measurements of the axes beforehand, the oval is laid out from an inscribed rectangle [34].

Pietro Cataneo (d. 1569) considered the oval in *L'architettura* (1567) (Book VII., Props. XII. XIII, XIIII and XIV). In *Come si causi la figura ovale, con il filo* (Prop. XIV) [35], Cataneo describes the laying out of an oval using the rope method, although he is, in fact, laying out an ellipse, and he is able to determine the measurements of the two main axes. In *Le Timon du Capitaine* (1587), Abroise Bachot (d.1587), proposed the continuous lay out of the ellipse with a rope and the invention of a tool for the delineation of ellipses (Bachot 1587). This work was reprinted and completed, and came to be known as *Le Gouvernail* [36]. The construction of ellipses by translating the middle of the major axis on the main axes is shown in plate 13 of *Traité des pratiques geometrales et perspectives* (1665) by Abraham Bosse (1602–1676) [37] and in Book II. Prop. XIII by Sébastien Le Clerc (1637–1714) of *Pratique de la géometrie, sur le papier et sur le terrain* (1669) [38] (Fig. 1).

Vicente Tosca (1651–1723) dedicated Volume III of the *Compendio mathematico* (1710) to Conical Sections, and he determined various designs in *De la elypse* [39]. He had previously considered the oval in Volume I (1707), *De la Geometría Práctica* (Prop. XIV–XVII). It is also mentioned in *Describir un óvalo, dados el mayor y menor diámetro*, specifically establishing the two main axes [40]. The solution was published in Volume V of the *Compendio* (1712), which covered civil and military architecture, Book II Prop. III [41]. The method makes the layout of ovals commensurable, which enables them to fit into two previously established axes, as happens in the drawing of the ellipse. However, Tosca's method involves the initial arbitrary setting of the radius of the minor arches such that infinite different ovals can be drawn, unlike the ellipse, which only has one solution.

The search for an approximation from the oval to the ellipse is determined by a method involving drawing the eight-centred oval in the *Liv. X Chap. II Des anses de Panier* by Charles-Étienne-Louis Camus (1699–1768) [42]. Honey determined *The eight-centred oval and ellipse* (1908) [43], in which he constructed an empirical and graphical eight-centred oval based upon the main axes. Further contributions to tracing the oval included the eight-centre method of Charles-Étienne-Louis Camus (1699–1768) Liv. X Chap. II Des anses de Panier, determining the *construction d'une anse a Panier à cinq centres*. Other methods were mentioned in treatises on construction, such as the one by Gustav Adolf Breymann (1807–1859) [44] or the treatises on industrial design by Thomas Ewing French (1871–1944) [45]. Similar methods for drawing eight-centred ovals that approximate to an ellipse were developed by Chaplin (1945) and Lockwood (1961).

It can be suggested that the laying out of the shape of an amphitheatre in the field depends upon two basic needs: the operational ease of its instrumental of drawing, and the degree of commensurability of the main axes.

The laying out of an amphitheatre is therefore determined by its geometric condition, which is linked to the technical instruments used by the Romans for the layout on the real scale: *gnomone, groma, lychnia*, and other *instrumentum mensoris* [46], or simply rope, *linea* (Etymologiarum XIX.18.3) [47]. Despite our relatively extensive knowledge of the geometry of the Roman world [48], and of the layout of the figures of the ellipse and the oval, our knowledge from direct sources is limited to the passage in Balbus Mensor (fl. c.100) [49]. However, the *groma* can be used to lay out the ellipse and the oval, and it could have been the basis for laying out the Coliseum [50].

The construction of the ellipse using a rope, *linea*, which was therefore continuous, is accredited to Anthemius of Tralles (c. 474–c. 558), Cataneo, and Bachot. The laying out of the ellipse from