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Joint optimization of number of wells, well locations and controls using a gradient-based algorithm

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ABSTRACT

This paper presents a detailed algorithm for solving the general well-placement optimization problem in which the number of wells, their locations and rates are simultaneously optimized with an efficient gradient-based algorithm. The proposed well-placement optimization algorithm begins by placing a large number of wells in the reservoir, where, the well rates are the optimization variables. During iterations of the algorithm, most of the wells are eliminated by setting their rates to zero. The remaining wells and their controls determine the optimal number of wells, their optimum locations and rates. The well-placement algorithm consists of two optimization stages. In the initialization stage, the appropriate total reservoir production rate (or the total injection rate) for the set of to-be-optimized producers (or injectors) is estimated by maximizing the net-present-value for the specified operational life of the reservoir. In the second stage, a modified net-present-value functional which also considers the drilling cost of the wells is maximized subject to the a total rate constraint determined in the initialization stage. Both stages of the algorithm use gradient projection to enforce the linear and bound constraints, where the required gradients are computed with the adjoint method. The bottomhole pressure constraints on the wells are enforced using a practical approach. The applicability and robustness of our well-placement algorithm is discussed through several example problems.

Keywords: Optimal well-placement; Well control optimization; Joint optimization; Adjoint gradient; Gradient projection; Bottomhole pressure constraint

1. Introduction

Determining the optimal number of the producers and injectors and their optimal locations is a critical step in preparing the development plan for a reservoir. The general well-placement optimization problem considers simultaneously optimizing the number of wells, well types, well locations and trajectories and well operating conditions for life cycle of the reservoir. To the best of our knowledge the general well-placement problem is far from solved, although many researchers have focused on solving individual components of the problem. Most papers on optimal well-placement assume the number of wells are fixed and the well operating conditions, wellbore pressures or rates, and the reservoir life are specified and fixed when optimizing well locations and trajectories (completions). We should mention, however, that in the optimization procedures used by Handels et al. (2007) and Emerick et al. (2009) the optimal number of wells may change by a very small number during the optimization process, and, in the optimization procedures proposed by Yeten et al. (2002) and Onwunalu and Durlofsky (2010), the number of laterals of a multi-lateral well is optimized during the optimization process. Beckner and Song (1995) proposed an algorithm to optimize the schedule of the wells (the time to bring the wells online) and with the algorithms proposed by Yeten et al. (2002), Emerick et al. (2009), Onwunalu and Durlofsky (2010) and Nwankwor et al. (2013) the types of the wells (injection or production) are also optimized. Recently, the joint optimization

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of well locations and well controls have been investigated by more researchers. Correspondingly, in the well-placement method developed by Yeten et al. (2002), the well controls are also optimized. Li and Jafarpour (2012) and Li et al. (2013) proposed well-placement optimization algorithms to optimize well locations and their rates simultaneously using stochastic gradients and Bellout et al. (2012) proposed a combined pattern search and sequential quadratic programming algorithm for joint optimization of well placement and controls. In this paper, we present the detailed algorithm of our method for solving a more general form of the well-placement problem, where the number of injection and production wells, their locations and their rates are optimized simultaneously.

Previously, most researchers parameterized the well-placement optimization problem in terms of discrete variables (the gridblock indices). Bittencourt and Horne (1997), Yeten et al. (2002), Ozdogan and Horne (2006), Lee et al. (2009) and Emerick et al. (2009) used the genetic algorithm and Centilmen et al. (1999) used simulated annealing to solve the resulting discrete optimization problem. To solve the discrete optimization problem more efficiently, Bangerth et al. (2006), Li and Jafarpour (2012) and Li et al. (2013) used versions of the simultaneous perturbation stochastic approximation (SPSA) algorithm. More recently, Onwunalu and Durlofsky (2010), Bouzarkouna et al. (2012), Forouzanfar et al. (2012) and Nwankwor et al. (2013) parameterized the well-placement optimization problem by representing the well trajectories inside the reservoir in terms of continuous real variables. Onwunalu and Durlofsky (2010) implemented particle swarm optimization (PSO) algorithm, Bouzarkouna et al. (2012) implemented "the covariance matrix adaptation evolution strategy" (CMA-ES) optimization method, Forouzanfar et al. (2012) implemented a derivative-free optimization algorithm based on the quadratic approximation of the objective function and Nwankwor et al. (2013) implemented a hybrid differential evolution and particle swarm optimization algorithm to solve their optimization problems.

Due to the promising computational efficiency superiority of gradient-based optimization methods over non-gradient-based optimization algorithms, some researchers tried to reformulate the optimal well-placement problem so that it can be solved with more efficient gradient-based algorithms; where, the required derivatives are efficiently computed by the adjoint method. Handels et al. (2007), Zandvliet (2008) and Sarma and Chen (2008) proposed methods for finding an improving direction for moving the vertical wells at each iteration of a gradient-based optimization algorithm. Vlemmix et al. (2009) developed an adjoint-based well trajectory optimization method which is effectively an extension of the Handels et al. (2007) idea to three dimensions. Wang et al. (2007), Zhang et al. (2010) and Forouzanfar et al. (2010) proposed a method for the optimization of the number of wells, their locations and well controls (rates) of the rate-controlled vertical wells. Forouzanfar et al. (2010) improved the basic idea of Wang et al. (2007) and Zhang et al. (2010) so it can be applied to more realistic problems.

In this paper, we modify and extend the well-placement method presented in Forouzanfar et al. (2010). In particular, the method of Forouzanfar et al. (2010) for setting bounds on flowing bottomhole pressure of the wells requires a robust method for determining the active upper bound constraints of the well controls. Here, we present a robust algorithm for determining active lower and upper bound constraints of the well controls at each optimization iteration. The resulting algorithm makes it possible to allow an eliminated well to reopen in future optimization iterations. This improves the robustness of our algorithm. In addition, the algorithm for the initialization step introduced in Forouzanfar et al. (2010) is modified to a more robust algorithm by introducing a new convergence criteria. Another contribution of this work is the design of a procedure that enables our well-placement optimization method to escape from a local maximum in order to obtain a set of local maxima for the problem, where, the best maximum represents the estimated optimal solution of the problem. Finally, we present the complete algorithm for our proposed well-placement method which explains every step of our algorithm in detail. In the results section, we present the results of our modified well-placement algorithm for a 2D synthetic and the PUNQ reservoir models.

2. Well-placement problem definition

The well-placement optimization algorithm presented in this paper, simultaneously estimates the optimal number of wells, well locations and operating conditions (well rates) for the life of the reservoir by maximizing a measure of the field net-present-value. For a reservoir under water flooding, the net-present-value (NPV) is defined as

$$\mathrm{NPV}=\sum_{n=1}^{N_t}\left[\frac{\sum_{j=1}^{N_{\mathrm{prd}}}\left(r_o q_{o,j}^n - r_w q_{w,j}^n\right)-\sum_{i=1}^{N_{\mathrm{inj}}}\left(r_{winj} q_{\mathrm{inj},i}^n\right)}{(1+b)^{t^n/365}}\right]\Delta t^n, \tag{1}$$

where N_t is the number of reservoir simulation time steps; N_{inj} and N_{prd}, are the number of injection and production wells, respectively; Δt^n represents the size of the nth timestep in days; t^n represents the total simulation time in days at the end of the nth timestep; $q_{o,j}^n$ and $q_{w,j}^n$, respectively, represent the average oil and water production rates of the jth producer over the nth simulation timestep and $q_{\mathrm{inj},i}^n$ is the average injection rate of the ith injection well over the nth simulation timestep; r_o in \$/STB is the oil revenue per unit volume; r_w in \$/STB is the water disposal cost per unit volume; r_{winj} in \$/STB is the water injection cost per unit volume and b is the annual discount rate.

Our well-placement optimization method requires that we start by a large number of wells that are approximately uniformly distributed throughout the reservoir. During the optimization algorithm, most of the wells will be eliminated by setting their rates to zero. The well-placement problem is defined as maximizing a modified net-present-value (NPV) functional, $J[\boldsymbol{u}]$, defined by

$$J(\boldsymbol{u}) = \mathrm{NPV} - \sum_{i=1}^{N_{\mathrm{inj}}}\left[f_{\mathrm{inj},i}(\boldsymbol{u})\,C_{\mathrm{inj}}\right] - \sum_{j=1}^{N_{\mathrm{prd}}}\left[f_{\mathrm{prd},j}(\boldsymbol{u})\,C_{\mathrm{prd}}\right], \tag{2}$$

where $\boldsymbol{u}$ denotes the vector of well controls (flow rates); NPV denotes the standard net-present-value function defined in Eq. (1); C_{inj} represents the cost of drilling one injection well and C_{prd} represents the cost of drilling one producing well; $f_{\mathrm{inj},i}(\boldsymbol{u})$ and $f_{\mathrm{prd},j}(\boldsymbol{u})$, respectively, are the drilling cost functions for the injection well i and the production well j, which are discussed in the following. In our well-placement optimization algorithm, we need to start with a large number of wells because no new wells can be added during the optimization process, i.e., the set of optimized well locations is always a