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Erratum

Separation of dispersed suspension in rotating test tube

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Abstract

An analytical model has been developed on separation, classification, and clarification of dispersed suspension with well-characterized particle size distribution using rotating tube centrifuge. The model is based on limiting trajectory analysis on individual particles of different sizes settling in the rotating tube under centrifugal acceleration with results expressed in a new dimensionless sedimentation Leung (*Le*) number, which corresponds with its counterpart for continuous-feed centrifuge. For sedimenting in a rotating tube the governing *Le* number depends on the effective viscosity of the suspension, the initial height of suspension, density difference between solid particles and suspension, centrifugal gravity, and the time duration that the suspension has been subject to centrifugation. *Le* is directly related to the cut size, therefore operating a centrifuge at condition with small *Le* number leads to improved separation.

In the first part of the model, centrifugal gravity *G* is assumed constant in the entire suspension domain wherein the effective radius for determining *G* is taken as the geometric mean of the smallest and largest radii of the suspension, and the sediment thickness is assumed negligible. The prediction from the model is compared with experimental measurements carried out in laboratory on four different suspensions. Laboratory measurements include (a) solids concentration, (b) particle size distribution, and (c) density, respectively, of feed and supernatant after subjecting test samples at different centrifugal gravities and time duration. Results of four laboratory test studies involving classification and separation of different suspension compared reasonably well with the model prediction. In two cases, the test results obtained from a small pilot continuous-feed centrifuge were compared with that of the rotating tube based on the same test sample.

In the second part, a variable-*G* model is also developed to account more accurately that the *G*-force increases linearly with radius across the suspension but also with negligible sediment thickness. The results confirmed that the constant-*G* model using the effective radius is reasonably accurate for engineering calculations.

In the third part, the effect on separation due to sediment accumulating over time at the tube bottom has been investigated for the constant-*G* model. The accumulating sediment shortens the particle settling distance; on the other hand it also undermines particles settling with clarified liquid flowing radially inward (opposing sedimentation) from displacement of liquid in sediment formation. The improved model with finite sediment thickness compares slightly more favorably with experimental results under condition of high-solids capture and high-rate of sedimentation at small *Le*.

Not only the rotating tube can be used to determine whether a given suspension is separable by centrifugation and also the pertinent parameters involved in scale-up associated with such laboratory testing, the present study addresses the feasibility of projecting results of process separation using continuous-feed centrifuges in absence of dynamic effects associated with complicated flow and sediment/cake solids transport. Pilot study using continuous-feed centrifuges is needed for demonstration and production scale-up and to further assess the impact based on any unforeseen complications.

Keywords: Rotating tube; Centrifuge; Sedimentation; Classification; Clarification; Sediment

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1. Introduction

Traditionally rotating-tube testing [1] is used in industrial laboratories to determine whether heavier solid particles in suspension are separable under centrifugal gravity, *G*, as realized by industrial centrifuges. This pertains to difficult-to-separate suspension with slowly settling solids, which under earth's gravity takes a long time (or required a large settling area) for sedimentation and clarification. Examples are solid particles with small density difference compared with the liquid suspension as found in bio-solids suspension and broth, or slurries containing finely dispersed micron-

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Nomenclature

A	defined by Eq. (C.9)
F_f	cumulative size under (%)
f_f	frequency of occurrence (%)
F_e	cumulative size under in centrate or supernatant (%)
G	centrifugal gravity (m/s^2)
g	gravity (m/s^2)
H	suspension height (m)
I	integral defined in Eq. (8c).
Le	Leung number defined by Eq. (7)
SR	size recovery defined by Eq. (11) (%)
R	radius (m)
R_b	radius at rotating tube bottom (m)
R_e	solids recovery in supernatant (%)
R_p	radius at suspension-air interface (m)
R_s	solids recovery in cake (%)
S	sediment thickness (m)
t	time (s)
u_p	velocity of particle in absolute/inertial frame (m/s)
u_L	velocity of liquid phase in absolute/inertial frame (m/s)
υ_s	settling velocity of particle relative to liquid (m/s)
W_e	supernatant solids concentration (% w/w)
W_f	feed solids concentration (% w/w)
W_s	cake solids concentration (% w/w)
x_c	maximum particle size in supernatant or cut size, micron
x_c'	maximum particle size in supernatant or cut size for finite sediment thickness, micron
x_k	particle size k, micron
x_o	reference particle size, such as mean or median size of a suspension, conveniently taken as one micron
Z_s	solids capture fraction in cake
Z_e	solids capture fraction in supernatant

Greek symbols

μ	suspension viscosity (cp)
μ'	viscosity factor (cp)
$\Delta\rho$	density difference between solids and suspension, kg/m^3
ρ_e	density of supernatant (kg/m^3)
ρ_f	density of feed (kg/m^3)
ρ_s	density of solids (kg/m^3)
ρ_L	density of liquid phase (kg/m^3)
Γ	dimensionless sediment parameter, [1]
λ	hindered settling function, [1]
ϕ	volume fraction of solids in suspension, [1]
ϕ_s	volume fraction of solids in sediment or cake, [1]
Φ	dimensionless group of solids volume fraction, Eq. (25c)
η	efficiency, [1]
ν	kinematic viscosity (m^2/s)
Ω	rotational speed (1/s)

Subscripts

a	acceleration
b	bottom of tube
e	supernatant or centrate
f	feed
k	particle size
L	liquid
o	reference
p	pool surface or particle
s	sediment/cake or solids

and even sub-micron-size particles in a viscous liquid or suspension.

The results of the centrifugation tests are usually expressed in terms of solids capture, or solids recovery, plotted as a function of different G for a fixed time duration t, or different time durations t for fixed G. Experiences [1] suggest that it is better to plot solids recovery versus the product Gt, rather than G or t holding other variables constant. If chemical flocculant or coagulant is used to agglomerate fine particles, the dosage of the chemical additive becomes another parameter in addition to Gt. It seems that there is a better rationale behind interpreting data obtained from rotating tube by examining the problem from first principle. Perhaps, one might want to get a projection, if possible, on performance of continuous-feed centrifuges based on properly conducted and analyzed rotating tube tests on a suspension sample. It is also a common experience rather than an exception that “representative” samples are limited especially on a new process, which lends itself suitably to small-scale laboratory testing such as with rotating tube. Furthermore, limited test runs with a small pilot centrifuge might be feasible with additional amount of sample. Frequently large-scale pilot testing would not happen until the process is further developed or when the plant is built subsequently. Therefore, it is of great interests not only to conduct, but to properly analyze and interpret rotating-tube test results of a given suspension sample.

In this paper, an analytical model has been developed to predict sedimentation of suspension with dispersed solids and particles in rotating tube. Previous studies [2,3] on related subject have not provided an adequate method to quantify and predict separation performance of rotating tube. Furthermore, the work [3] is limited to zone settling for highly agglomerated solids and assumed further the settling velocity is related to the solids volume fraction by an empirical