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Cessation of Couette and Poiseuille flows of a Bingham plastic and finite stopping times

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Abstract

We solve the one-dimensional cessation Couette and Poiseuille flows of a Bingham plastic using the regularized constitutive equation proposed by Papanastasiou and employing finite elements in space and a fully implicit scheme in time. The numerical calculations confirm previous theoretical findings that the stopping times are finite when the yield stress is nonzero. The decay of the volumetric flow rate, which is exponential in the Newtonian case, is accelerated and eventually becomes linear as the yield stress is increased. In all flows studied, the calculated stopping times are just below the theoretical upper bounds, which indicates that the latter are tight.
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1. Introduction

In viscometric flows, one can bring a fluid to a halt by setting the moving boundary to rest in the case of Couette flows, or by reducing the applied pressure gradient to zero in Poiseuille flows. In a Newtonian fluid, the corresponding steady velocity fields decay to zero in an infinite amount of time [1]. In a Bingham plastic, the velocity fields go to zero in a finite time, which emphasizes the role of the yield stress [2,3]. Glowinski [2] and Huilgol et al. [3] have provided explicit theoretical finite upper bounds on the time for a Bingham material to come to rest in various flows, such as the plane and circular Couette flows and the plane and axisymmetric Poiseuille flows. To be specific, each upper bound depends on the density, the viscosity, the yield stress and the least eigenvalue of the Laplacian operator on the flow domain [2,3]. As for the underlying cause for the finite extinction time, it can be shown that the yield surface moves laterally with a finite speed bringing the fluid to a halt, and that kinematical conditions play a crucial role [4]. In a similar fashion, the upper bounds derived by Huilgol [5] for the cessation of axisymmetric Poiseuille flows with more general viscoplastic fluids must be caused by the lateral movement of the yield surface.

The objective of the present work is to compute numerically the stopping times and make comparisons with the theoretical upper bounds provided in the literature for the cessation of three flows of a Bingham fluid: (a) the plane Couette flow; (b) the plane Poiseuille flow; (c) the axisymmetric Poiseuille flow. Instead of the ideal Bingham-plastic constitutive equation, we employ the regularized equation proposed by Papanastasiou [6], to avoid the determination of the yielded and unyielded regions in the flow domain. It should be noted that preliminary results for the case of the plane Poiseuille flow can also be found in Ref. [7].

The paper is organized as follows. In Section 2, we discuss the regularized Papanastasiou equation for a Bingham plastic. In Section 3, we present the dimensionless forms of the governing equations for the three flows of interest along with the corresponding theoretical upper bounds. In Section 4, we present and discuss representative numerical results for all flows. The numerical stopping times are just below

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the theoretical upper bounds, i.e. the latter are tight. Some discrepancies are observed only for low Bingham numbers when the growth parameter in the Papanastasiou model is not sufficiently high. Finally, Section 5 contains the conclusions of this work.

2. Constitutive equation

Let $\mathbf{u}$ and $\boldsymbol{\tau}$ denote the velocity vector and the stress tensor, respectively, and $\dot{\boldsymbol{\gamma}}$ denote the rate-of-strain tensor,

$$\dot{\boldsymbol{\gamma}} \equiv \nabla\mathbf{u} + (\nabla\mathbf{u})^{\mathrm{T}}, \tag{1}$$

where $\nabla\mathbf{u}$ is the velocity-gradient tensor, and the superscript T denotes its transpose. The magnitudes of $\dot{\boldsymbol{\gamma}}$ and $\boldsymbol{\tau}$ are respectively defined as follows:

$$\dot{\gamma} = \sqrt{\frac{1}{2}\mathrm{II}_{\dot{\boldsymbol{\gamma}}}} = \sqrt{\frac{1}{2}\dot{\boldsymbol{\gamma}} : \dot{\boldsymbol{\gamma}}} \quad \text{and} \quad \tau = \sqrt{\frac{1}{2}\mathrm{II}_{\boldsymbol{\tau}}} = \sqrt{\frac{1}{2}\boldsymbol{\tau} : \boldsymbol{\tau}}, \tag{2}$$

where II stands for the second invariant of a tensor.

In tensorial form, the Bingham model is written as follows:

$$\begin{cases} \dot{\boldsymbol{\gamma}} = \mathbf{0}, & \tau \leq \tau_0, \\ \boldsymbol{\tau} = \left(\frac{\tau_0}{\dot{\gamma}} + \mu\right)\dot{\boldsymbol{\gamma}}, & \tau \geq \tau_0, \end{cases} \tag{3}$$

where τ_0 is the yield stress, and μ is a constant viscosity.

In any flow of a Bingham plastic, determination of the yielded ($\tau \geq \tau_0$) and unyielded ($\tau \leq \tau_0$) regions in the flow field is necessary, which leads to considerable computational difficulties in the use of the model. These are overcome by using the regularized constitutive equation proposed by Papanastasiou [6]:

$$\boldsymbol{\tau} = \left\{\frac{\tau_0[1-\exp(-m\,\dot{\gamma})]}{\dot{\gamma}} + \mu\right\}\dot{\boldsymbol{\gamma}}, \tag{4}$$

where m is a stress growth exponent. For sufficiently large values of the regularization parameter m, the Papanastasiou model provides a satisfactory approximation of the Bingham model, while at the same time the need of determining the yielded and the unyielded regions is eliminated. The model has been used with great success in solving various steady and time-dependent flows (see, for example, [8,9] and the references therein).

3. Flow problems and governing equations

The governing equations along with the boundary and initial conditions of the three time-dependent, one-dimensional Bingham-plastic flows of interest are discussed below. The theoretical upper bounds of Glowinski [2] and Huilgol et al. [3] for the stopping times are also presented.

3.1. Cessation of plane Couette flow

The geometry of the plane Couette flow is shown in Fig. 1a. The steady-state solution is given by

$$u_x^s(y) = \left(1 - \frac{y}{H}\right)V, \tag{5}$$

where V is the speed of the lower plate (the upper one is kept fixed) and H is the distance between the two plates. We assume that at $t = 0$, the velocity $u_x(y, t)$ is given by the above profile and that at $t = 0^+$ the lower plate stops moving. To nondimensionalize the x-momentum equation, we scale the lengths by H, the velocity by V, the stress components by $\mu V/H$, and the time by $\rho H^2/\mu$, where ρ is the constant density of the fluid. With these scalings, the x-momentum equation becomes

$$\frac{\partial u_x}{\partial t} = \frac{\partial \tau_{yx}}{\partial y}. \tag{6}$$

The dimensionless form of the Papanastasiou model is reduced to

$$\tau_{yx} = \left\{\frac{Bn[1-\exp(-M\dot{\gamma})]}{\dot{\gamma}} + 1\right\}\frac{\partial u_x}{\partial y}, \tag{7}$$

where $\dot{\gamma} = |\partial u_x/\partial y|$,

$$Bn \equiv \frac{\tau_0 H}{\mu V} \tag{8}$$

is the Bingham number, and

$$M \equiv \frac{mV}{H} \tag{9}$$

is the dimensionless growth parameter.

The dimensionless boundary and initial conditions are as follows:

$$\begin{aligned} &u_x(0,t) = 0, \quad t > 0, \qquad u_x(1,t) = 0, \quad t \geq 0, \\ &u_x(y,0) = 1 - y, \quad 0 \leq y \leq 1. \end{aligned} \tag{10}$$

In the case of a Newtonian fluid ($Bn = 0$), the analytical solution of the time-dependent flow, governed by Eqs. (6), (7) and (10), is known [1]:

$$u_x(y,t) = \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{1}{k}\sin(k\pi y)\,\mathrm{e}^{-k^2\pi^2 t}. \tag{11}$$

Hence, the flow ceases theoretically in an infinite amount of time. If the fluid is a Bingham plastic ($Bn > 0$), however, the flow comes to rest in a finite amount of time, as demonstrated by Huilgol et al. [3], who provide the following upper bound for the dimensionless stopping time:

$$T_f \leq \frac{4}{\pi^2}\ln\left[1 + \frac{\pi^2}{2}\frac{\|u_x(y,0)\|}{Bn}\right], \tag{12}$$

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