J. Non-Newtonian Fluid Mech. 129 (2005) 128–142

Journal of Non-Newtonian Fluid Mechanics

www.elsevier.com/locate/jnnfm

Review of chaos in the dynamics and rheology of suspensions of orientable particles in simple shear flow subject to an external periodic force

K. Asokan[a], C.V. Anil Kumar[b], J. Dasan[c], K. Radhakrishnan[d], K. Satheesh Kumar[e], T.R. Ramamohan[f,*]

[a] *Govternment Polytechnic College, Attingal 695101, Kerala, India*
[b] *St. John's College, Anchal, Kerala, India*
[c] *College of Engineering, Trivandrum 695016, Kerala, India*
[d] *N.S.S. College, Nilamel 691535, Kerala, India*
[e] *Department of Physics, Pohang University of Science and Technology, Pohang 790784, Republic of Korea*
[f] *CSIR Centre for Mathematical Modelling and Computer Simulation, Bangalore 560037, India*

Received 7 September 2004; received in revised form 31 March 2005; accepted 17 June 2005

Abstract

We review results obtained over a period of about a decade on a class of technologically and fundamentally important problems in suspension rheology viz., the dynamics and rheology of dipolar suspensions of orientable particles in simple shear flow. The areas explored in this review include effects such as the fluid flow field, external forcing, Brownian diffusion, hydrodynamic interactions and their impact on the rheological properties of the suspension. The main feature of the presentation is the use of a uniform framework in which one or more of the above effects can be studied, based on Langevin type equations for particle orientations combined with a brute-force technique for computing orientational averages. These models are capable of capturing complex dynamical behaviour in the system such as the presence of subharmonics or chaos, both in the dynamics and rheology. The tools developed allow for investigating how chaos in the system is affected by Brownian diffusion and hydrodynamic interactions. The presence of chaos opens up a number of novel possibilities for dynamical and rheological behaviour of the system, which can be put to efficient use in many ways, e.g. in separating particles by aspect ratio and possibly developing computer controlled intelligent rheology. The results also have implications for certain areas of chaos theory, such as a new intermittency route to chaos and the possibility of non-trivial collective behaviour in spatially extended systems. These studies highlight certain deficiencies in current techniques in the literature for handling the rheology of dilute and semi-dilute suspensions. In the presence of Brownian motion the proposed method computes the averages by simulating a set of deterministic ordinary differential equations rather than stochastic differential equations. The systems considered may also serve as a paradigm for analysing how microscopic chaotic fluctuations in spatially extended systems affect macroscopic averages. We also attempt to put our results into context with respect to recent work on rheochaos in complex fluids such as liquid crystals and nematic polymers.

PACS: 05.45.–a; 05.70.Fh

Keywords: Suspension; Particles; Rheology; Chaos; Diffusion; Brownian; Langevin; Interaction

1. Introduction

A systematic theoretical study of suspensions of particles in fluids can be considered to have started with the seminal work of Einstein [1] on Brownian motion namely, the

* Corresponding author.
E-mail address: trr432@yahoo.com (T.R. Ramamohan).

0377-0257/$ – see front matter
doi:10.1016/j.jnnfm.2005.06.003

Nomenclature

b	equatorial radius of a spheroid in the suspension
c_1, c_2	see Eq. (8)
C	$=(r^2-1)/(r^2+1)$, a shape factor for the particle
D_r	diffusion coefficient
E	rate of deformation tensor
$F(\boldsymbol{x}, \boldsymbol{y})$	function of $j_0(\boldsymbol{x})$ (see Eq. (11))
$G(\boldsymbol{q})$	Green's function for the hydrodynamic interaction corresponding to orientation $\boldsymbol{q}$
$G'(\boldsymbol{q})$	$= G(\boldsymbol{q}) - \mathsf{J}\boldsymbol{q}$
$H(\boldsymbol{x}, \boldsymbol{y})$	certain function of $j_1(\boldsymbol{x})$ (see Eq. (11))
$\boldsymbol{i}$	unit vector along the x-direction
$j_0(\boldsymbol{x}), j_1(\boldsymbol{x})$	spherical Bessel functions of zeroth and first orders
$\boldsymbol{k}$	the scaled dimensionless form of the external force
k_1, k_2, k_3	Cartesian components of $\boldsymbol{k}$
l	half the length of a fiber
n	number of particles per unit volume
P	a constant depending on the shape of the particle (see Eq. (2))
Pe	Péclet number ($=\dot{\gamma}/D_r$)
$\mathsf{J}\boldsymbol{q}$	Oseen tensor for orientation $\boldsymbol{q}$
Q	a constant depending on the shape of the particle (see Eq. (2))
Q_1, Q_2	functions of F and H (see Eq. (10))
r	aspect ratio of the particle
R	a constant depending on the shape of the particle (see Eq. (2))
$\boldsymbol{u}$	unit vector indicating the orientation of the particle
$\boldsymbol{u}^J$	Jeffery's rotation rate of particle
u_1, u_2, u_3	Cartesian components of $\boldsymbol{u}$
$\boldsymbol{v}$	undisturbed velocity profile of the flow
y	the y-coordinate

Greek letters

$\dot{\gamma}$	shear rate
Γ	Gaussian white noise vector
$\Gamma_1, \Gamma_2, \Gamma_3$	Cartesian components of $\boldsymbol{\Gamma}$
$\boldsymbol{\delta}$	unit tensor
$[\eta_1], [\eta_2]$	the apparent viscosities
θ	polar angle in spherical co-ordinates
ξ_i	component of the orientation vector $\boldsymbol{q}$ in the direction $\boldsymbol{n}_i$
$[\tau_1]$	the first normal stress difference
$[\tau_2]$	the second normal stress difference
ϕ	azimuthal angle in spherical co-ordinates
Φ	the volume fraction of the particles in the fluid
$\psi(\boldsymbol{u}, t)$	particle orientation distribution function
ω	frequency of the external force driver
$\boldsymbol{\Omega}$	vorticity vector

irregular and random fluctuations of tiny particles suspended in a fluid, first noticed by the botanist Robert Brown while observing a suspension of pollen grains. Einstein provided an explanation for Brownian motion based on kinetic theory and proceeded to estimate the effective viscosity of the suspension. There are a number of practical situations where suspensions of a more general nature than those studied by Einstein have to be considered. This warrants considering additional effects that can influence both the dynamics of individual particles and bulk suspension properties. For example, if the suspending medium is sheared, the resulting flow field influences both the local orientations of the particles and their relative positions in the medium. Hydrodynamic interactions among particles, caused by the disturbance that the presence of each particle produces on its neighbours, is another factor that becomes important in concentrated solutions with larger volume fraction of particles. Interactions are, however, neglected in sufficiently dilute suspensions which are characterised by the limit $nl^3 \ll 1$ where n is the number of particles per unit volume and l is the linear dimension of the particles. If the particles have electric or magnetic charges, as in ferromagnetic particles immersed in a ferrofluid, an external electric or magnetic field can affect the local microstructure. The shape of the particles is also important; for example, non-spheres have definite orientation effects unlike spheres, and spheroids usually behave differently from triaxial ellipsoids. A complete knowledge of the rheology of suspensions, subjected to one or more of these effects, is desirable in the manufacturing of suspensions of industrial importance. Determining rheological properties through experiments is prone to many artefacts and instrumental errors which are difficult to remove, particularly when the particle size is small. Numerical simulation of rheological properties is often useful since errors caused by the deficiency of measurement techniques or unavoidable defects in the material or flow conditions can be minimized in simulations; but this requires the model and the simulation algorithm to be fairly accurate. Theoretical models for suspensions which take care of the various orientational effects are, therefore, important, both from a theoretical and practical point of view and help in developing intuition about such problems. There are a number of studies in the literature incorporating one or more of these factors, but most of the theoretical work is in dilute suspensions in Newtonian fluids.

The starting point of most of these investigations is the classic work of Jeffery [2] that describes the creeping motion of rigid, non-Brownian spheroids in an incompressible, Newtonian fluid subjected to a simple shear flow. According to