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An overview of approximation methods for large-scale dynamical systems ☆

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Abstract

Methods for the approximation of large-scale dynamical systems will be surveyed. There are mainly two families namely, the SVD-based and Krylov-based approximation methods. The former family is based on the singular value decomposition and the second on moment matching. While the former has many desirable properties including an error bound, it cannot be applied to systems of high complexity. The strength of the latter on the other hand, is that it can be implemented iteratively and is thus appropriate for application to high complexity systems. An effort to combine the best attributes of these two families leads to a third class of approximation methods, which will be referred to as SVD/Krylov. Following a survey of these methods we will conclude with a new result concerning model reduction with preservation of passivity which is appropriate for application to large-scale circuits arising in VLSI chip performance verification.

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1. Introduction

Starting point is a dynamical system which has to be simulated and/or eventually controlled. The first step in this endeavor consists of modeling, that is deriving equations describing its behavior. In our setting these will be assumed to be differential equations, either partial (PDEs) or ordinary (ODEs). To proceed with their solution, the PDEs are often discretized in space which leads to a large number of ODEs. Assume for simplicity that these consist of a set of $n$coupled first order ODEs. Model reduction consists in replacing them with k coupled first order ODEs where $k \ll n$; in addition the reduced set of ODEs must behave as closely as possible to the original one. A pictorial representation of how model reduction fits in the overall picture of simulation/control is shown in Fig. 1.

For details on the material presented in the sequel we refer to the book (Antoulas, 2005a).

1.1. Problem statement

We will consider dynamical systems described by (explicit) state equations

$$\boldsymbol{\Sigma} : \dot{x} = f(x,u), \quad y = h(x,u),$$

with state $x(\cdot)$of dimension n, input $u(\cdot)$ of dimension m, and output $y(\cdot)$ of dimension p, where $n \gg m, p$; we will use the notation $\boldsymbol{\Sigma} = (f, h)$. The approximation or model reduction problem can be formulated as follows.

Problem: Approximate $\boldsymbol{\Sigma} = (f, h)$ with $\hat{\boldsymbol{\Sigma}} = (\hat{f}, \hat{h})$, $u(\cdot) \in \mathbb{R}^m, \hat{x}(\cdot) \in \mathbb{R}^k, \hat{y}(\cdot) \in \mathbb{R}^p$, where $k \ll n$, so that as many as possible of the properties below are satisfied:

(1) Approximation error small and existence of an error bound.
(2) Preservation of stability/passivity.
(3) Procedure must be computationally efficient.

1.2. Approximation by projection

The approximation methods to be discussed are obtained by means of *projections*. Let $V, W \in \mathbb{R}^{n\times k}$, be such that $W^*V = I_k$, where the superscript $(\cdot)^*$ denotes transposition. It follows that $\Pi = VW^*$ is a projection. Let $\hat{x} = W^*x \in \mathbb{R}^k$; the state x will

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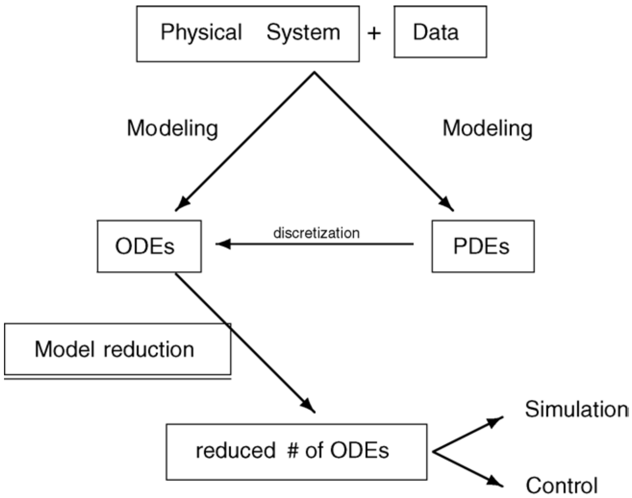

Fig. 1. Model reduction: the big picture.

now be approximated by its projection $\Pi x = V\hat{x}$. This leads to the following system which describes the evolution of $\hat{x}$:

$$\hat{\boldsymbol{\Sigma}} : \dot{\hat{x}} = W^* f(V\hat{x}, u), \qquad \hat{y} = h(V\hat{x}, u).$$

Thus $\hat{\boldsymbol{\Sigma}}$ is "good" approximation of $\boldsymbol{\Sigma}$, if $x - \Pi x$ is "small" in some appropriate sense.

1.2.1. Special case: linear dynamical systems

In this case f and h are linear, i.e., $\dot{x} = Ax + Bu, y = Cx + Du$, and the system will be denoted by

$$\boldsymbol{\Sigma} = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right) \in \mathbb{R}^{(n+p)\times(n+m)}.$$

With V, W as above, a $k < n$ dimensional reduced order model $\hat{\boldsymbol{\Sigma}}$: $\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u, \hat{y} = \hat{C}\hat{x} + Du$, of $\boldsymbol{\Sigma}$ is obtained as follows:

$$\hat{\boldsymbol{\Sigma}} = \left(\begin{array}{c|c} \hat{A} & \hat{B} \\ \hline \hat{C} & D \end{array}\right) = \left(\begin{array}{c|c} W^*AV & W^*B \\ \hline CV & D \end{array}\right) \in \mathbb{R}^{(k+p)\times(k+m)}.$$

The closeness of $\hat{\boldsymbol{\Sigma}}$ to $\boldsymbol{\Sigma}$ will be measured mostly in terms of the $\mathcal{H}_\infty$ and the $\mathcal{H}_2$ norms. The former is the 2-norm of the worst output error $\|y - \hat{y}\|_2$ for inputs $\|u\|_2 = 1$, while the latter is the 2-norm of the difference between the corresponding impulse responses $\|h - \hat{h}\|_2$.

2. Motivating examples

There is a great variety of examples which motivate the need for model reduction of large-scale systems, both for simulation and control. A partial list is given in Table 1.

In the sequel we will summarize a few of these applications; for a complete discussion we refer to the book (Antoulas, 2005a).

2.1. Passive devices: VLSI circuits

Evolution of VLSI design. The integrated circuit (IC) was invented in the 1960's. In 1971 the Intel 4004 processor was introduced. It had 2300 components of size approximately 10μm and an operating frequency of 64 KHz. Thirty years later, in 2001, the Intel Pentium IV was introduced, having 42 million components with size of the order of 0.18μm and 2 GHz operating frequency; as a result the *interconnect length*(length of all interconnections between the components) is about 2 km, and the chip has seven layers. In this case the interconnections must be modeled as transmission lines and simulations are required to verify that internal electromagnetic fields do not significantly delay or distort circuit signals. This leads to the need for electromagnetic modeling of interconnects (and packages). The resulting models are very complex; using PEEC (Partial Element Equivalent Circuit) methods to discretize Maxwell's equations in three-dimensions, we obtain systems of complexity $n \approx 10^5 \cdots 10^6$; thus model reduction methods are necessary to verify the performance of the underlying chip.

For details see e.g. van der Meijs (2000).

Table 1
Motivating examples of large-scale systems

1. *Passive devices*:
VLSI circuits
2. *Weather prediction, data assimilation*:
North sea forecast
Air quality forecast
America's cup
3. *Biological systems*:
Honey comb vibrations
4. *Molecular systems*:
Dynamics simulations
Heat capacity
5. *ISS: International space station*:
Stabilization
6. *Vibration/acoustic problems*:
Windscreen vibrations
7. *CVD reactor*:
Bifurcations
8. *Optimal cooling*:
Steel profile
9. *MEMS: Micro-electro-mechanical systems*:
Elf sensor

2.2. Weather: wave surge forecast

This problem concerns the prediction of wave surge at the coast of the Netherlands; in cases of high waves certain dams need to be closed to prevent flooding.

The North Sea is shallow compared, say, to the Atlantic ocean and therefore wave propagation is governed by the *shallow water equations*. A typical discretization results in about 60,000 ODEs.The computational time (on a laptop) is about 2 days which is prohibitively long. Therefore the system needs to be reduced.

Actually, there are eight measurement stations which provide weather data, and hence the resulting problem is one of *data assimilation*. Thus a Kalman Filter is needed, and the reduced filter propagates low-rank factors of the data covariance matrix.

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