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Control of plant-wide systems using dynamic supply rates[☆]

Michael J. Tippett, Jie Bao [1]

School of Chemical Engineering, The University of New South Wales UNSW, Sydney, NSW 2052, Australia

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ABSTRACT

This paper presents a framework for analysis of plant-wide processes from a network perspective. Using the concept of dissipativity, the conditions for plant-wide input–output stability and performance are developed, based on the dissipativity of individual subsystems and the topology of the network of the plant-wide process. Dynamic supply rates, expressed as quadratic differential forms, are proposed not only to render dissipativity based analysis less conservative but also allow the dynamic plant-wide performance criteria to be specified in terms of desired closed loop supply rates. The links between the plant-wide supply rate, finite $\mathcal{L}_2$ gain in an extended input–output space and weighted $\mathcal{H}_\infty$ norm are explored in this paper. These results lay a foundation for a supply rate-centric approach to plant-wide distributed control.

1. Introduction

Largely driven by increasingly tight economic and environmental requirements, modern chemical plants are becoming increasingly complex, often with dozens of process units with material recycling and energy integration. From a process control point of view, recycling streams can be understood as positive feedback loops within the process network, which have a deleterious impact on control performance (Luyben, Tyréus, & Luyben, 1998). These strong interactions between process units are a key feature of plant-wide process control problems, and are a challenge to control practice due to high sensitivity to disturbances and possible plant-wide instability (Kumar & Daoutidis, 2002). Another important challenge in plant-wide process control is the scale of the problem, which can make centralized control systems computationally difficult or infeasible (Skogestad, 2004).

Some existing approaches for control of plant-wide processes deal with the interactions between unit processes as uncertainties, as presented in Grosdidier and Morari (1986), Samyudia, Lee, and Cameron (1994), Swarnakar, Marquez, and Chen (2009) and Skogestad and Morari (1989). A decentralized controller is then designed to stabilize the plant-wide system subject to these uncertainties based on robust control theory. This approach can greatly simplify implementation of the control system. To help facilitate this, interaction measures for plant-wide process systems have been developed, e.g. Bristol (1966), Cheng and Li (2010) and Manousiouthakis, Savage, and Arkun (1989). These interaction measures are useful for input–output pairing and estimating the efficacy of decentralized control schemes. However, this approach is inherently conservative as the known interactions are treated as unknown.

Distributed control structures have gained attention in recent years. For example, an approach requiring minimal communication between controllers is presented in Goodwin, Haimovich, Quevedo, and Welsh (2004); the optimal controller network topology is determined in Langbort and Gupta (2009); distributed estimation and model predictive control methods are developed in Mercangöz and Doyle (2007) and Vadigepalli and Doyle (2003). Distributed model predictive control has received much attention recently, e.g. Liu, Chen, Muñoz de la Peña, and Christofides (2010) and Stewart, Venkat, Rawlings, Wright, and Pannocchia (2010).

One promising approach to plant-wide analysis is based on a network perspective, decomposing a plant-wide process into individual process units interacting through a network with a known topology (e.g., Bao, Jillson, & Ydstie, 2007; Rojas, Setiawan, Bao, & Lee, 2009; Ydstie, 2002). A key advantage of this approach is its *scalability*, as it allows analysis and control design based on simpler subsystems and their interconnections.

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E-mail addresses: michael.tippett@unsw.edu.au (M.J. Tippett), j.bao@unsw.edu.au (J. Bao).

1 Tel.: +61 2 9385 6755; fax: +61 2 9385 5966.

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Dissipativity theory, introduced in Willems (1972), has become an important analysis and control design tool. It is particularly suitable for the network approach to plant-wide control, as the plant-wide stability and performance can be determined based on the dissipativity of individual subsystems and the network topology, thus simplifying control design. A dynamical system with input $u \in \mathbb{R}^p$, output $y \in \mathbb{R}^q$ (both with compact support) and state $x \in \mathbb{R}^n$ respectively is said to be dissipative if there exists a function defined on the input and output variables, called the supply rate $s(u, y)$ and positive semidefinite (at least once differentiable) function defined on the state space, called the storage function $V(x) \geq 0$, such that

$$\dot{V}(x(t)) \leq s(u(t), y(t)). \quad (1)$$

The following quadratic (Q, S, R)-type supply rate is commonly used:

$$s(u, y) = y^T Q y + 2y^T S u + u^T R u, \quad (2)$$

where $Q = Q^T$, S and $R = R^T$ are real matrices.

Results on the application of dissipativity theory to large-scale systems analysis and decentralized control design has been reported in the literature. For example, in Moylan and Hill (1978), Rojas, Setiawan, Bao, and Lee (2008) and Vidyasagar (1981) where dissipativity is used as an enabling tool for stability and operability analysis of large-scale systems. In Hangos, Alonso, Perkins, and Ydstie (1999), a special case of dissipativity, passivity, is used to show the stability of plant-wide nonlinear process systems. This is achieved using the links between passivity and the mass and energy balances underlying process systems. These links were developed in Alonso and Ydstie (1996) and Ydstie and Alonso (1997), where thermodynamics is used to develop physically motivated storage functions. These concepts are used for plant-wide control in Farschman, Viswanath, and Ydstie (1998). However, the above passivity/dissipativity approaches can be very conservative, as the constant supply rates shown in (2) can only provide a coarse description of the systems dynamic features.

This work presents a new approach to plant-wide stability and performance analysis and design based on network decomposition and the dissipativity of the subsystems. Both the plant-wide stability and performance specifications (in the form of weighted $\mathcal{H}_\infty$ norms) are achieved by encoding them as desired closed loop supply rates. Controllers can then be designed to achieve these supply rates, and hence, the desired closed-loop stability and performance properties. Central to the proposed approach is the use of dynamic supply rates, expressed as quadratic differential forms (QDFs). As the QDFs represent a more general form of dissipativity (Pendharkar & Pillai, 2008; Willems & Trentelman, 1998), they capture more detailed dynamic properties and therefore lead to far less conservative dissipativity based conditions than those based on constant supply rates (e.g., in (2)), as shown in Tippett and Bao (2013a). The QDFs also allow the dynamic plant-wide performance criteria to be specified in terms of desired closed loop supply rates as was done in our previous work (Tippett & Bao, 2013b). In the proposed approach, the models of individual process units are only used to validate their dissipativity. All analysis and control design is performed based on the dissipativity of the process units rather than detailed process models. This leads to a supply rate-centric approach to plant-wide distributed control that can deal with *arbitrary* process network topologies and control structures.

The remainder of this paper is structured as follows. In Section 2, a brief overview of dissipativity theory is presented. A dissipativity based decomposition of the plant-wide system and controller network is presented in Section 3. In Section 4 plant-wide stability and performance results based on this decomposition are presented.

2. Dissipativity, stability and performance

The key concept used in this paper is dissipativity, which is defined in the context of behavioral systems theory as follows:

Definition 1 (*Willems & Trentelman, 1998*)**.** A controllable system is said to be dissipative with storage function Q_ψ, and supply rate Q_ϕ, if:

$$\int_{-\infty}^{\infty} Q_\phi(w)\, \mathrm{d}t \geq Q_\psi(w), \quad (3)$$

for all allowable trajectories of the system with compact support, with $w = \left(y^T\ u^T\right)^T$, a vector of the inputs and outputs.

In this paper we are concerned with the case where Q_ψ, the storage function, is positive semidefinite. This is equivalent to half-line dissipativity on $\mathbb{R}_-$ (Willems & Trentelman, 1998). Note that half-line dissipativity on $\mathbb{R}_-$ implies dissipativity in the general sense in of Definition 1. In this context Q_ϕ and Q_ψ are quadratic differential forms as shown below:

$$Q_\phi(w) = \sum_{k=0}^{k_{\max}} \sum_{l=0}^{l_{\max}} \left(\frac{\mathrm{d}^k}{\mathrm{d}t^k} w\right)^T \phi_{kl} \left(\frac{\mathrm{d}^l}{\mathrm{d}t^l} w\right), \quad (4)$$

where ϕ_{kl} are constant, symmetric, matrices and are the coefficient matrices of the two-variable polynomial matrix $\phi(\zeta, \eta)$. The indeterminates ζ and η refer to $\frac{\mathrm{d}}{\mathrm{d}t}^T$ and $\frac{\mathrm{d}}{\mathrm{d}t}$ respectively. $k_{\max}$ and $l_{\max}$ are the highest order of $\frac{\mathrm{d}}{\mathrm{d}t}^T$ and $\frac{\mathrm{d}}{\mathrm{d}t}$. The degree of the supply rate is defined as $\max(k_{\max}, l_{\max})$. The matrix $\phi(\zeta, \eta)$ is said to *induce* the QDF (Willems & Trentelman, 1998).

Definition 2 (*Willems & Trentelman, 1998*)**.** Let $\phi \in \mathbb{R}^{q\times q}[\zeta, \eta]$, with ϕ symmetric. The QDF $Q_\phi(w)$ is called positive, denoted by $\phi > 0$, if $Q_\phi(w) \geq 0$ for all w, and the only w for which $Q_\phi(w) = 0$ is $w = 0$. A QDF is negative definite, $\phi < 0$, if and only if $-\phi > 0$.

It worth pointing out that the derivative a of QDF is itself a QDF, i.e., $\frac{\mathrm{d}}{\mathrm{d}t} Q_\phi = Q_{\frac{\mathrm{d}}{\mathrm{d}t}\phi}$, where $\frac{\mathrm{d}}{\mathrm{d}t}\phi = (\zeta + \eta)\phi$. In simple terms, QDFs are quadratic functions of the inputs and outputs and a finite number of their derivatives. They can be understood as an extension of the commonly used (Q, S, R) shown in (2) that include derivative terms. Because of this, QDF supply rates capture more detailed system information than constant supply rates, allowing for more in depth analysis and less conservative results. The extended variables of input u and output y will be used throughout this paper and are defined as follows:

$$\hat{y} = \left(y^T \quad \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^T \quad \dots \quad \left(\frac{\mathrm{d}^{\tilde{n}} y}{\mathrm{d}t^{\tilde{n}}}\right)^T\right)^T$$

$$\hat{u} = \left(u^T \quad \left(\frac{\mathrm{d}u}{\mathrm{d}t}\right)^T \quad \dots \quad \left(\frac{\mathrm{d}^{\tilde{m}} u}{\mathrm{d}t^{\tilde{m}}}\right)^T\right)^T \quad (5)$$

for some integers $\tilde{n}$ and $\tilde{m}$ less than or equal to the order of the system in y and u respectively. Using these variables, the quadratic from (4) can be rewritten as:

$$Q_\phi\begin{pmatrix} y \\ u \end{pmatrix} = \begin{pmatrix} \hat{y} \\ \hat{u} \end{pmatrix}^T \tilde{\phi} \begin{pmatrix} \hat{y} \\ \hat{u} \end{pmatrix} = \begin{pmatrix} \hat{y} \\ \hat{u} \end{pmatrix}^T \begin{pmatrix} \tilde{Q} & \tilde{S} \\ \tilde{S}^T & \tilde{R} \end{pmatrix} \begin{pmatrix} \hat{y} \\ \hat{u} \end{pmatrix}. \quad (6)$$

The block matrix $\tilde{\phi}$ is referred to as the coefficient matrix of ϕ, as its entries are the constant coefficient matrices of the polynomials of the indeterminates ζ and η in $\phi(\zeta, \eta)$. Methods for determining dissipativity of linear systems with QDF supply rates and storage functions include Pick matrix and frequency domain conditions presented in Willems and Trentelman (1998) and a Linear