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A distributed continuous-time algorithm for network localization using angle-of-arrival information ☆

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ABSTRACT

This paper studies the angle-of-arrival (AOA) localization problem, namely, localizing networks based on the angle-of-arrival measurements between certain neighboring network nodes together with the absolute locations of some anchor nodes. We propose the concepts of stiffness matrix and fixability for the anchored formation graphs modeling the networks, and show that they provide a complete characterization of the AOA localizability as well as an explicit formula for the localization result. Moreover, a distributed continuous-time algorithm is proposed that converges globally to the correct localization result on fixable formation graphs. Performances of the proposed algorithm, e.g., convergence rate and robustness to communication delay, are characterized and optimized. Sensitivities of the localization results with respect to errors in AOA measurements and anchor node positions are also analyzed.

1. Introduction

A multi-agent system consisting of a group of sensors, robots, vehicles, etc., can collaboratively accomplish tasks that are difficult or even infeasible for any individual agent. Examples include a network of sensors monitoring the occurrence of forest wildfires (Bodrozic, Stipanicev & Stula, 2006), a team of autonomous underwater vehicles mapping seabed terrains (Leonard et al., 2007), and a group of robots fetching a large object (Mellinger, Shomin, Michael, & Kumar, 2013). A crucial task in these applications is to find the (possibly time-varying) locations of all the agents based on sensor measurement data available to the agents. A direct localization method is to use positional devices such as GPS. However, due to issues such as cost, energy usage, and form factor, such devices are typically available only at a subset of the agents. To localize other agents, further inter-agent measurement data is needed. Designing algorithms that only utilize local measurements to localize the whole multi-agent network has become a popular research topic (Patwari et al., 2005).

Based on the type of measurement data available, localization algorithms can be classified into two categories: distance-based schemes and direction-based schemes. In the distance-based schemes, the relative distances between certain neighboring agents are available for localization purposes. Numerous distance-based localization algorithms have been proposed in the literature using, for example, received signal strength (RSS) (Savarese, Rabaey, & Beutel, 2001), time of arrival (TOA) or time difference of arrival (TDOA) (Savvides, Han, & Strivastava, 2001), or a combination of RF and ultrasound sensing (Priyantha, Chakraborty, & Balakrishnan, 2000). A common issue with these algorithms however is that eliminating ambiguity, especially reflective ambiguity, is often very difficult, and may result in large errors (Priyantha, Balakrishnan, Demaine, & Teller, 2003), even after significant simplifications (Moore, Leonard, Rus, & Teller, 2004). Indeed, the distance-based localization problem has been shown to be NP-hard (Dieudonne, Labbani-Igbida, & Petit, 2010; Saxe, 1979), with a unique solution existing only when the underlying graph is globally rigid (Eren et al., 2004). From a practical point of view, distance measurements also requires the (often infeasible) knowledge of the signal transmission characteristics for RSS-based methods, and sophisticated time synchronization mechanism for TOA/TDOA-based methods.

In comparison, the direction-based localization schemes use angle-of-arrival (AOA) measurements instead of relative distances. Such schemes have not received as much attention mainly due to two drawbacks. First, the AOA localization problem is also NP-hard (Bruck, Gao, & Jiang, 2009). However, if compasses are installed on all the agents to allow for a common global coordinate for the measured angles, the direction-based localization problem could be as easy as solving a set of linear equations (Ash & Potter, 2007; Eren, Whiteley, & Belhumeur, 2006). Second, measuring AOA of RF signals requires advanced hardware such as antenna arrays

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that are usually expensive and large in size due to wavelength constraints. Nevertheless, in many environmental monitoring applications, the acoustic sensing devices (Chen, Yao, & Hudson, 2002) that the agents/sensors are equipped with provide an inexpensive means for acquiring AOA information (Patwari et al., 2005). As the technologies advance, these drawbacks could be alleviated to a point where AOA localization schemes become a viable alternative or at least a valuable supplement to distance-based schemes.

In this paper, we first formulate the AOA localization problem within the framework of formation graph theory, which is an extension of the classical graph theory by incorporating the positional information of the vertices. It is shown that the AOA localization problem is equivalent to finding the solution to a system of linear equations (see also Ash & Potter, 2007; Eren et al., 2006). Instead of a centralized solution that needs the inversion of a large-scale matrix, we propose a distributed AOA localization algorithm whose unique globally asymptotically stable equilibrium is the desired localization result. The convergence rate and delay tolerance of the proposed algorithm are also analyzed and optimized through the solution of a condition number optimization problem. The algorithm is then extended to the case of networks with switching topologies.

Compared to existing approaches (e.g. Ash & Potter, 2007; Eren et al., 2006) that characterize AOA localizability using rank conditions on the rigidity matrix, our method utilizes the stiffness matrix first proposed in Zhu and Hu (2009) and characterizes AOA localizability using the novel concept of graph fixability. One advantage of our method is that the stiffness matrix is positive semidefinite; its eigenvalues provide quantitative measures of AOA localizability. Another advantage is that, since the stiffness matrix has a similar structure to that of the celebrated Laplacian matrix arising in the study of consensus problems (Ren, Beard, & Atkins, 2005), distributed AOA localization algorithms can be inspired from consensus algorithms (e.g. Olfati-Saber & Murray, 2004; Ren et al., 2005).

This paper is organized as follows. The concept of formation graphs and their properties are reviewed in Section 2. In Section 3, the AOA localization problem is formulated, and AOA localizability is characterized by the fixability of the underlying formation graph. In Section 4, we propose a distributed continuous-time AOA localization algorithm, and analyze its performance. A problem of optimizing the algorithm's performance is also formulated and solved in Section 4. In Section 5, the proposed algorithm is extended to networks with switching topologies. Localization errors caused by inaccurate measurements are analyzed in Section 6. We conclude this paper and give some prospective questions that deserve further research in Section 7. Supplementary proofs are provided in Appendices.

1.1. Notation

For a symmetric matrix A, we write $A \succeq 0$ if A is positive semidefinite. For $\mathbf{v} = [a\, b]^\top \in \mathbb{R}^2$, $\angle\mathbf{v}$ denotes the principal value of argument within the range $[0, 2\pi)$ of the complex number $a+bi \in \mathbb{C}$; and $\mathbf{v}^\perp = [-b \;\; a]^\top \in \mathbb{R}^2$ denotes the 90° counterclockwise rotation of $\mathbf{v}$. Let $Q : \mathbb{R}^2 \to \mathbb{R}^2$ be the operator such that $Q : \mathbf{v} \mapsto \mathbf{v}^\perp$ and $Q^{-1} : \mathbf{v} \mapsto -\mathbf{v}^\perp$. If $\mathbf{p}$ is a stacked vector with $\mathbf{p}_i \in \mathbb{R}^2$ as its components, then we use $\mathbf{p}^\perp$ to denote the vector with components $\mathbf{p}_i^\perp$. Denote by $\mathbb{SO}_n \triangleq \left\{T \in \mathbb{R}^{n\times n} : T^\top T = I_n, \det(T) = 1\right\}$ the set of all n-dimensional special orthogonal matrices.

2. Formation graphs and rigidity

In this section, the concepts of (anchored) formation graphs and some of their properties will be reviewed. These provide the theoretical framework for studying the localization problem in Section 3.

Definition 1 (*Formation Graph*). A formation graph is a triple $(\mathcal{V}, \mathbf{p}, K)$ consisting of the following.

- $\mathcal{V} = \{1, 2, \dots, n\}$ is the index set of n vertices (agents, sensor nodes, etc.) on the plane.
- $\mathbf{p} = \begin{bmatrix}\mathbf{p}_1^\top & \mathbf{p}_2^\top & \cdots & \mathbf{p}_n^\top\end{bmatrix}^\top \in \mathbb{R}^{2n}$ is the (position) *configuration* of the n vertices, where $\mathbf{p}_i \in \mathbb{R}^2$ denotes the position of vertex i. We assume that $\mathbf{p}_i \neq \mathbf{p}_j$ for $i \neq j$.
- $K = \left[k_{ij}\right]_{i,j\in\mathcal{V}} \in \mathbb{R}^{n\times n}$ is the *connectivity matrix*, where k_{ij} is the *connectivity coefficient* between vertices $i, j \in \mathcal{V}$, and it satisfies $k_{ii} = 0$, $k_{ij} \geq 0$, and $k_{ij} = k_{ji}$.

Definition 2 (*Anchored Formation Graph*). A formation graph $(\mathcal{V}, \mathbf{p}, K)$ together with a nonempty anchor set $\mathcal{A} \subset \mathcal{V}$ is called an *anchored formation graph*, and is denoted by $(\mathcal{V}, \mathbf{p}, K, \mathcal{A})$. Vertices in $\mathcal{A}$ and $\mathcal{F} \triangleq \mathcal{V} \setminus \mathcal{A}$ are called *anchors* and *free vertices*, respectively.

In the applications of network localization and formation control, anchors usually refer to those nodes that know their absolute locations via, for example, positioning devices.

Definition 3 (*Consistency*). Two formation graphs $(\mathcal{V}, \mathbf{p}, K)$ and $(\mathcal{V}, \mathbf{p}', K)$ with the same vertex set and connectivity matrix are called *consistent* if $\|\mathbf{p}_i - \mathbf{p}_j\| = \|\mathbf{p}'_i - \mathbf{p}'_j\|$ for all those $i, j \in \mathcal{V}$ with $k_{ij} > 0$.

Likewise, two anchored formation graphs $(\mathcal{V}, \mathbf{p}, K, \mathcal{A})$ and $(\mathcal{V}, \mathbf{p}', K, \mathcal{A}')$ are *consistent* if (i) $(\mathcal{V}, \mathbf{p}, K)$ and $(\mathcal{V}, \mathbf{p}', K)$ are consistent; (ii) $\mathcal{A} = \mathcal{A}'$; and (iii) $\mathbf{p}_i = \mathbf{p}'_i$ for all $i \in \mathcal{A}$.

Two vertices $i, j \in \mathcal{V}$ are called *connected* if their connectivity coefficient $k_{ij} > 0$. Consistent formation graphs have the same relative distances between connected vertices. For anchored formations, consistency requires in addition the same set of anchors and anchor positions.

If two formation graphs have the same relative distances between *all* pairs of vertices, not just connected ones, then there exists a congruent transformation composed of rotations, translations, and reflections that transforms the vertex positions of one to the other; thus we have the following definition.

Definition 4 (*Congruency*). Two formation graphs $(\mathcal{V}, \mathbf{p}, K)$ and $(\mathcal{V}, \mathbf{p}', K)$ are *congruent* if $\|\mathbf{p}_i - \mathbf{p}_j\| = \|\mathbf{p}'_i - \mathbf{p}'_j\|$ for all $i, j \in \mathcal{V}$. Two anchored formation graphs $(\mathcal{V}, \mathbf{p}, K, \mathcal{A})$ and $(\mathcal{V}, \mathbf{p}', K, \mathcal{A}')$ are *congruent* if $(\mathcal{V}, \mathbf{p}, K)$ is congruent to $(\mathcal{V}, \mathbf{p}', K)$, $\mathcal{A} = \mathcal{A}'$ and $\mathbf{p}_i = \mathbf{p}'_i$ for all $i \in \mathcal{A}$.

Obviously, congruent formation graphs are consistent. The reverse, however, does not hold in general, except for formation graphs possessing the following property.

Definition 5 (*Global Rigidity (Eren et al., 2004)*). A formation graph $(\mathcal{V}, \mathbf{p}, K)$ is called *globally rigid* if any formation graph $(\mathcal{V}, \mathbf{p}', K)$ consistent with it must also be congruent to it.

In essence, global rigidity characterizes the inflexibility of the shape of the formation, given that the relative distances between connected vertex pairs are kept constant. In the distance-based localization problem, exact localization is possible only for globally rigid formation graphs (Eren et al., 2004).

Determining global rigidity is, however, in general difficult. An easier alternative is its infinitesimal version. For a given formation graph $(\mathcal{V}, \mathbf{p}, K)$, the distance constraints between connected vertices can be equivalently summarized as $k_{ij}\|\mathbf{p}_j - \mathbf{p}_i\|^2 \equiv k_{ij}d_{ij}^2$