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Leader tracking in homogeneous vehicle platoons with broadcast delays[☆]

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ABSTRACT

For vehicle platoons, the leader following control architecture is known to be capable of achieving string stability while maintaining tight formations. In this paper, we study a variety of schemes where the leader state is available to the other members of the platoon. We show that in some cases it is possible to achieve string stability in the presence of certain amounts of time delay in the leader state reception. We also compare other properties of the different schemes and discuss some of their advantages and disadvantages.

1. Introduction

In recent decades, formation control of autonomous vehicles has received great attention; see for example Chien and Ioannou (1992), Chu (1974), Levine and Athans (1966) and Swaroop and Hedrick (1996). More recently, researchers have studied extensively the simple case of a 1-D platoon of vehicles with linear dynamics, considering diverse alternatives to achieve coordinated movement of the string (see for example Hao, Yin, & Kan, 2012; Jovanovic & Bamieh, 2005; Lin, Fardad, & Jovanovic, 2012, and the references therein).

One simple approach, which can be implemented using linear controllers, is to equip every member of the formation with a compensator that stabilizes its position in closed loop, using as a reference the position of its predecessor in the string and a desired constant inter-vehicle spacing. The internal stability of the whole system is ensured by the simple interconnection, the design of the compensator and the assumption of linear dynamics for each vehicle. In Seiler, Pant, and Hedrick (2004) it was shown that this simplistic architecture suffers from a drawback known as "string instability" namely, the amplification of disturbances along the string as a response to a disturbance in a single vehicle. In this case, the problem occurs if identical vehicles and local controllers are used (homogeneous control) and whenever the controller-vehicle pair has two integrators, regardless of the chosen compensator parameters. Moreover, in Barooah and Hespanha (2005) the authors show that including also the immediate follower position in the control signal of each vehicle (bidirectional control) does not remove the disturbance amplification, complementing the work done in Seiler et al. (2004).

The term "string stability" has been defined in many different ways. In this work we consider a similar approach to that in Middleton and Braslavsky (2010): in an interconnection of multiple systems, we consider a set of relevant closed loop transfer functions (e.g. the ones that describe the effect of disturbance on the inter-vehicle spacings). String stability occurs if the functions have frequency magnitude peaks that are bounded independently of the platoon size. String *instability*, has a number of undesirable implications for the safety and performance of a platoon of vehicles, the most dramatic being the increased chance of collisions as the size of the platoon grows. Several measures aimed at ensuring string stability of a formation have been proposed. The authors of Chien and Ioannou (1992), Klinge and Middleton (2009) and Swaroop, Hedrick, Chien, and Ioannou (1994) introduce "time headway",

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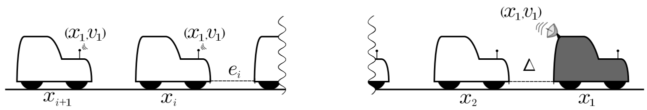

Fig. 1. Platoon of vehicles.

where inter-vehicle spacings are dependent on vehicle velocities. Equipping each vehicle with a controller that depends on its position along the string (heterogeneous control), has also been proposed to overcome the difficulty, see for example Khatir and Davidson (2004), Lestas and Vinnicombe (2007) and Shaw and Hedrick (2007). Unfortunately this only helps if control "bandwidths" are allowed to diverge (either to 0 or ∞) as the string length grows, (Middleton & Braslavsky, 2010). Alternatively, "leader following" schemes such as the ones studied in Seiler et al. (2004) and Xiao, Gao, and Wang (2009) obtain string stability of the formation by providing every follower with the state (or an estimate) of the leader, at the cost of increasing networking requirements. A limited range of forward communication, which would imply a loss in leader state reception along the string, does not allow linear controllers to achieve string stability without the use of constant time headway policies (see Middleton and Braslavsky (2010)). The use of a network to provide the members of a string of vehicles with the leader state (position and/or velocity) immediately poses questions on the effect of disruptions of the communication. In this context, the works presented in Liu, Goldsmith, Mahal, and Hedrick (2001) and Xiao et al. (2009) studied the effect of time delays for leader following schemes under restrictions on controller structure.

Other researchers have studied more complex approaches to formation control that also present issues as the platoon size increases. In Jovanovic and Bamieh (2005), the authors study optimal control strategies for platoons with an increasing number of vehicles and show that some related LQR problems are "ill-posed". The work in Bamieh, Jovanovic, Mitra, and Patterson (2012) shows that in the 1-D case it is impossible to have large "coherent" platoons with only local feedback. A PDE approach was used by the authors of Barooah, Mehta, and Hespanha (2009) to show that the "stability margin" of a bidirectional control architecture can be improved by "mistuning". More recently Lin et al. (2012) integrated the previous results in the design of optimal controllers to enhance the coherence of a formation.

Leader following schemes can provide string stability and "tight formations" in which a fixed, prescribed inter-vehicle spacing is maintained, regardless of the platoon speed. These properties ensure some degree of safety and performance for the vehicles' manoeuvres and are important for applications where the tightness of the formations is required (e.g., increased throughput in Automated Highway Systems, Hedrick, Tomizuka, & Varaiya, 1994).

The main contributions of this paper can be summarized as follows. We revisit the leader–predecessor following architecture where each follower tracks simultaneously the positions of its immediate predecessor and the leader. We also propose two novel alternatives based on it: the first one considers a modification of the way the leader position is communicated to the followers; the second one makes the followers track the velocity of the leader instead of its position. We provide formulae in the frequency domain for the dynamics of the resulting interconnections with and without the presence of time delays and for general linear controllers. Finally, we study the string stability properties of the three architectures mentioned above and their ability to achieve a tight formation.

The paper is organized as follows. Section 2 gives the notation and framework. Section 3 defines the control architectures to be studied and in Section 4 we present the corresponding vehicle dynamics. The main results of the paper are presented in Section 5. Section 6 includes numerical examples that illustrate the results obtained and Section 7 gives some final remarks.

2. Framework and problem formulation

2.1. Notation

The notation used in this paper follows much of the standard systems and control literature. Lowercase is used for real scalar signals, $x : \mathbb{R} \to \mathbb{R}$ with specific values of the signal denoted by $x(t)$. Uppercase is used for scalar complex-valued Laplace transforms of signals and transfer functions, $X : \mathbb{C} \to \mathbb{C}$ with specific values denoted by $X(s)$. For the sake of brevity in the notation, where there is no confusion, the argument (s) will be omitted. Vectors will be denoted as $\underline{x}(t) \in \mathbb{R}^n$ and $\underline{X} \in \mathbb{C}^n$, while $\underline{x}(t)^\top$ and $\underline{X}^\top$ denote their transposes. The imaginary unit is denoted by j, with $j^2 = -1$. Boldface will be used for matrices $\boldsymbol{G} \in \mathbb{C}^{n\times m}$ and the (i, k)th entry of $\boldsymbol{G}$ is denoted by $G_{i,k}$. The magnitude of X when $s = j\omega, \omega \in \mathbb{R}$, is denoted by $|X|$ and its magnitude peak over all possible values of ω is denoted as $\|X\|_\infty := \sup_\omega |X(j\omega)|$. For $z \in \mathbb{C}$, $\Re(z)$ and $\Im(z)$ denote the real and imaginary parts of z respectively.

2.2. Vehicle model and preliminaries

We consider a platoon of $n \in \mathbb{N}$ identical vehicles that travel in a straight line, with the aim of maintaining a desired and constant inter-vehicle spacing $\Delta > 0$. The vehicle dynamics considered are linear and time invariant, namely

$$m\ddot{x}_i(t) = -k_d\dot{x}_i(t) + d_i(t) + f(u_i(t)) \quad \text{for } 1 \le i \le n, \tag{1}$$

where: $x_i(t)$ denotes the position of the ith vehicle along the string; m its mass; k_d is the vehicle drag coefficient; f is the force applied by the engine, which is a function of $u_i(t)$, the control signal; and $d_i(t)$ is a disturbance force that acts on the vehicle. Assuming simple dynamics for the engine, i.e. $f(u_i(t)) = u_i(t)$, we can work in the frequency domain. For simplicity we assume that every car starts from rest and is initially positioned in the desired formation, that is $x_i(0) = (1 - i)\Delta$ and $\dot{x}_i(0) = 0$ for $i = 1, \dots, n$ and we define $\tilde{x}_i(t) = x_i(t) + (i - 1)\Delta$. Therefore, taking the Laplace transform we obtain the frequency domain vehicle models

$$\tilde{X}_i = \frac{U_i + D_i}{s(ms + k_d)} = X_i + (i - 1)\frac{\Delta}{s} \quad \text{for } 1 \le i \le n. \tag{2}$$

Now, the control goal is to keep a tight formation, that is, to maintain the errors $e_i^{pre}(t) = x_{i-1}(t) - x_i(t) - \Delta = \tilde{x}_{i-1}(t) - \tilde{x}_i(t)$ as close to zero as possible. This small error performance should be achieved in steady state, under disturbances to any member of the platoon, and for a constant speed of the leader. To achieve this, the control signal for each vehicle $u_i(t)$ is computed using the local measurement of the immediate predecessor position (indirectly through the measure of the inter-vehicle spacing) and the information being received from the leader (see Fig. 1). As a consequence, the leader–follower errors $e_i^{lea}(t) = x_1(t) - x_i(t) - (i - 1)\Delta = \tilde{x}_1(t) - \tilde{x}_i(t)$ will have a steady state response similar to that of $e_i^{pre}(t)$. These error signals can be associated with the performance of the system when considering traffic density issues and throughput. With this, we have that the Laplace transforms for the errors are given by

$$E_i^{pre} = X_{i-1} - X_i - \frac{\Delta}{s} = \tilde{X}_{i-1} - \tilde{X}_i, \tag{3}$$

$$E_i^{lea} = X_1 - X_i - (i - 1)\frac{\Delta}{s} = \tilde{X}_1 - \tilde{X}_i. \tag{4}$$

These two errors will be central to our later analysis.