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Brief paper

On the robust control of stable minimum phase plants with large uncertainty in a time constant. A fractional-order control approach[☆]

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ABSTRACT

This paper addresses the problem of designing controllers that are robust to a great uncertainty in a time constant of the plant. Plants must be represented by minimum phase rational transfer functions of an arbitrary order. The design specifications are: (1) a phase margin for the nominal plant, (2) a gain crossover frequency for the nominal plant, (3) zero steady state error to step commands, and (4) a constant phase margin for all the possible values of the time constant (T): $0 < T < \infty$. We propose a theorem that defines the structure of the set of controllers that fulfil these specifications and show that it is necessary for these robust controllers to include a fractional-order *PI* term. Examples are developed for both stable and unstable plants, and the results are compared with a standard *PI* controller and a robust controller designed using the *QFT* methodology.

1. Introduction

This article studies control systems that are robust to large uncertainties in a time constant. We have designed controllers that preserve the value of the phase margin. This allows us to approximately preserve: (a) the system damping, and (b) some robustness features, signifying that small changes in other parameters do not excessively degrade the performance of the closed-loop system.

Examples of systems whose linear models often undergo changes in one of their time constants as a consequence of their operating regimes – signifying that this time constant cannot be accurately determined and thus preventing a proper tuning of the controller – are: DC-motors with a variant electrical or mechanical time constant owing to changes in temperature, machining force processes in metal cutting when the depth-of-cut increases, electrical circuits in which the value of the resistance or capacitance of its elements may vary as a result of strong environmental changes, high pressure flow recycling systems powered by pumps or compressors, etc. What is more, many complex systems have a dominant real pole whose variation is a main concern, while variations of the secondary poles are considered to have little influence on the dynamics.

Various techniques that allow robust closed-loop systems to be obtained have been developed on the basis of the frequency response. The most popular are the H_∞, and the *QFT* methods. These are well suited to the design of robust controllers for plants that exhibit bounded uncertainties, but they experience difficulties in managing plants that undergo extreme variations in some parameters—and consequently exhibit large parameter uncertainties.

One of the first works on the design of control systems that are robust to great uncertainties in a plant parameter was carried out by Bode, who in 1945 studied the feedback amplifier design (Bode, 1945) and found that the optimal number of stages, as regards maintaining the phase margin constant (relative stability) when the amplifier gain undergoes great changes, is non-integer. This led to an open-loop transfer function of the form $G(s) = K/s^\alpha,\ \alpha \in \Re$, which exhibits a constant phase in a broad frequency interval (flat phase diagram) around the gain crossover frequency. Changes in the system gain therefore modify the gain crossover frequency but the phase margin is preserved. Oustaloup (1991) used this idea as a basis to develop a methodology with which to design robust control systems using fractional-order controllers: the *CRONE* method. Three generations of *CRONE* controllers have been developed. The first and second generations use algebraic methods to obtain the open-loop "Bode's ideal transfer function". The third *CRONE* generation method (Lanusse, Oustaloup, & Mathieu, 1993) deals with model uncertainties other than the gain, and attains robustness by minimizing a cost related to the variation of the closed-loop system damping.

Preserving a phase margin when plant parameters are uncertain implies that damping, or step input response overshoot,

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remains approximately constant. This iso-damping feature has been designed for the case of system gain uncertainties by imposing the local property that the derivative of the phase with regard to the frequency must be zero at the gain crossover frequency, i.e., the local phase flatness property. This property allows only limited parameter uncertainties. Standard *PID* controllers were designed with this property in Chen and Moore (2005), in addition to fractional-order *PI* controllers (Monje, Vinagre, Chen, & Feliu, 2004), and fractional-order *PID* controllers (Monje, Vinagre, Feliu, & Chen, 2008).

Controllers with predefined structures were designed to achieve a nominal phase margin and gain crossover frequency, and the local phase flatness robustness property. For example, fractional-order controllers with the structures $K_p + K_i/s^\alpha$ and $(K_p + K_s s)^\alpha$ were designed in Luo and Chen (2009) for the case of an integrator plus a fractional-order pole plant, and a methodology with which to design controllers of the form $K_p + K_i/s^\alpha$ for a first order plus time delay plant was presented in Luo and Chen (2012). Moreover, the phase margin was preserved for the robust motion control of a first-order plus time delay in series with an integrator plant, which exhibited a time constant with limited variation, by using a controller of the form $(K_p + K_s s)^\alpha$ (Jin, Chen, & Xue, 2011).

This article defines the family of controllers that "perfectly" preserve the phase margin in the case of stable minimum phase plants of an arbitrary order, subject to an arbitrarily large uncertainty in one plant time constant. It additionally shows that these controllers can be used in unstable plants. Unlike the three previous works, no "a priori" structure is imposed on the controller (of either an integer or fractional order nature), and the "flat phase condition" is imposed globally for all the frequencies rather than locally, thus permitting arbitrary time constant variations.

The paper is organized as follows. Section 2 presents the robust control design problem. Section 3 develops the main result of this article. Section 4 develops two illustrative examples and Section 5 states some conclusions.

2. Problem statement

Let us consider a single input–single output linear plant with the transfer function $G(s, T)$:

$$G(s,T) = \frac{1}{1+sT}\hat{G}(s), \tag{1}$$

where T is the unknown time constant that takes a value in the interval $0 \le T < \infty$, and $\hat{G}(s)$ is a stable and minimum phase rational transfer function which represents the well determined part of the plant. The nominal plant $G(s, T_0)$ corresponds to the nominal value T_0 of the variable time constant.

Let us then consider a unity feedback control scheme whose feedforward path consists of a generic controller, $R(s)$, in series with the plant, $G(s, T)$. The controller can be designed using three specifications (see Jin et al., 2011):

(1) Phase margin ϕ for the nominal plant $G(s, T_0)$.
(2) Gain crossover frequency ω_{c0} for the nominal plant $G(s, T_0)$.
(3) Robustness condition to changes in the time constant:

$$\left.\frac{d\mathrm{Arg}(G(j\omega,T)R(j\omega))}{d\omega}\right|_{\omega=\omega_{c0},T=T_0} = 0. \tag{2}$$

The first two specifications for the nominal plant can be expressed in a compact form by using the complex equation:

$$-e^{j\phi} = \frac{\hat{G}(j\omega_{c0})}{1+j\omega_{c0}T_0}R(j\omega_{c0}). \tag{3}$$

The third specification (2) is a "local" robustness condition which pursues a robust phase margin property for limited parameter variations. From here on this robustness property will be referred to as a *local isophase margin condition*. In this article, this "local" condition is substituted by the following "global" isophase margin condition.

Global Isophase Margin Criterion. The closed-loop system (G, R) is robust to uncertainties in parameter T in the interval $T \in [0, \infty)$, in the sense that its phase margin remains constant if condition

$$-e^{j\phi} = \frac{\hat{G}(j\omega_c)}{1+j\omega_c T}R(j\omega_c) \tag{4}$$

is verified in the entire range of variation of T. In this expression, ω_c is the gain crossover frequency of $G(s, T)R(s)$ that corresponds to a value T.

The objective of this article is, therefore, not only to tune the parameters of controller $R(s)$ but *to determine the structure of the controller that fulfils the constant phase margin robustness condition (4).*

Condition (4) involves two variables: T and ω_c. They are not independent because a given value of the time constant defines a gain crossover frequency value by solving Eq. (4). Let us define this relation by using the function $T = f(\omega_c)$, and denote it as the "$T - \omega_c$ function".

3. The main result

Assuming that controller $R(s)$ is known, function f can be obtained by means of (4). It can be numerically attained by solving (4) for given values of ω_c. But this procedure has the drawback that if $R(s)$ does not have the right structure then the resulting values of T will be complex numbers, which do not have any physical meaning. The family of controllers $R(s)$ that verify (4) *must therefore yield a real relation (real function) between the varying time constant and its corresponding gain crossover frequency* ω_c. This function must verify the nominal condition

$$T_0 = f(\omega_{c0}). \tag{5}$$

Controllers verifying (4) are provided in the following theorem.

Theorem. *Consider a plant of the form (1) whose varying time constant may take values in the interval* $0 \le T < \infty$. *The controller* $R(s)$ *of lowest order which:*

(1) *preserves a desired phase margin* ϕ $(0 < \phi < 90^\circ)$ *of* $G(s, T)R(s)$ *in the entire range of variation of* T,
(2) *yields a desired gain crossover frequency value* ω_{c0} *at a particular time constant value* T_0, *and*
(3) *exhibits zero steady state error to step commands*

consists of the series connection of a fractional-order PI controller (hereafter denoted as the FPI controller) and the inverse of the well determined part of the dynamics of the plant. This controller is given by:

$$R(s) = \underbrace{\left(\left(\frac{\tan(\phi)+\omega_{c0}T_0}{\omega_{c0}^{\alpha}}\right)s^{\alpha} - \frac{1}{\cos(\phi)}\right)}_{FPI\ controller}\hat{G}^{-1}(s), \tag{6}$$

where α *is a negative value:*

$$\alpha = \frac{2}{\pi}\phi - 1 \tag{7}$$

and the "$T - \omega_c$ *function" that describes the relationship yielded by this controller between the time constant and the gain crossover*