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Brief paper

A partial parameterization of nonlinear output feedback controllers for saturated linear systems[☆]

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ABSTRACT

This paper addresses the stabilization problem of linear systems subject to input saturation. The major purpose is to introduce nonlinearity into control laws so as to expand the design freedom for performance enhancement. To this end, a new approach is developed which involves a partial differential matrix inequality (PDMI). A class of stabilizing feedback laws is explicitly obtained by solving this PDMI analytically and the feedback laws are parameterized by a nonlinear function. Further, it is revealed that any linear observer can be used to realize the output feedback stabilization. Numerical examples, including the seek control of a hard disk drive, show that the introduced nonlinearity does contribute to the improvement of system performance. The application to integral control is also discussed.

1. Introduction

Linear systems with input saturation are commonly encountered in practice due to the inherent constraint on actuators. The dynamic performance of systems is severely constrained by the limitation of actuators. For example, in power systems the saturation of the magnetic excitor poses the greatest challenge to the transient stability of power generators (Kundur, 1994). The study of such systems has received great attention because of its importance in engineering practice. For this class of systems, it is difficult to achieve high performance only by linear feedback control due to the input saturation. In fact, it is well accepted that nonlinear control generally outperforms linear control for such systems.

Overview of existing methods

Historically, the control of saturated systems was first discussed in the context of integrator windup phenomena. The well-known classic antiwindup method is to reduce the integrator gain through an inner loop with a dead-zone compensation when the actuator is saturated (Hanus, Kinnaert, & Henrotte, 1987). With the progress in robust control and nonlinear control, the recent trend is either to maximize the region of stability (for unstable plants) or to optimize the control performance. The tools mostly used are passivity, LPV and LMI.

Roughly speaking, the design philosophy falls into two categories. (i) Design a low gain controller so as to avoid actuator saturation. The small-gain approach, such as Teel (1996), is along this line. A notable feature of this approach is that the potential of the actuator is not effectively used. As a consequence, it is difficult to achieve high performance. (ii) Actively make use of actuators and compensate the effect of saturation. The famous conditioning technique (Hanus et al., 1987) and most of the recent literature are in this direction.

The key in saturation control is how to utilize the characteristic of the actuator saturation in system design. There are two major approaches that make use of different aspects of the saturation. (i) Antiwindup approach: in this approach, a linear controller is designed without consideration of input saturation and $u_c - u_s$, the difference between the input command u_c and the output u_s of the actuator, is used to compensate the effect of actuator saturation (Hanus et al., 1987). In recent years, the antiwindup compensator has been extended to include dynamics and the feedback controller and antiwindup compensator are designed simultaneously (Grimm et al., 2003; Hu, Teel, & Zaccarian, 2009; Kothare, Campo, Morari, & Nett, 1994; Mulder, Kothare, & Morari, 2001). Tarbouriech and Turner (2009) is a good source on this topic which includes an extensive list of literature. (ii) Direct approach: the passivity and/or sector property of the saturation is used

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directly in control design, such as the Lyapunov approach (Chen, Lee, Peng, & Venkataramanan, 2003; Hu & Lin, 2003).

From the viewpoint of control structure, most of the modern antiwindup methods use a linear feedback controller and linear antiwindup compensator. LMI is used as an optimization tool either to enlarge the region of stability or to achieve some induced $\mathcal{L}_2$ norm specification. Particularly, Grimm et al. (2003, Facts 2, 3 in Sec. V) succeeded in combining the induced $\mathcal{L}_2$ norm specification with the passivity of dead-zone nonlinearity nicely via the well-known S-procedure. Meanwhile, in the composite approach of Hu and Lin (2003) and Chen et al. (2003), some type of nonlinear controllers are proposed to enhance control performance. In the aspect of global stabilization of saturated systems, there have been many new developments. Sussmann, Sontag, and Yang (1994) proposed a nested design technique with a structure similar to the neural network. Teel (1995) proposed a scheduled H_∞-type control method by scheduling a parameter according to the size of the state. A design technique is proposed in Lin (1998) which schedules both the low-gain and the high-gain of the control law. Zhou, Lin, and Duan (2010) extended this gain scheduling approach and addressed the implementation issue. Moreover, Tyan and Berstein (1999) discussed the linear control for systems containing a double integrator, based on a Lur'e–Postnikov type Lyapunov function. A detailed comparison of many saturation control methods is conducted in Rao and Bernstein (2001) with respect to the double integrator.

Objective and contribution of this paper

This paper pursues the nonlinear control of saturated systems from a different angle. The engineering motivation is that nonlinear control is able to achieve some wonderful performance, such as finite-time settling and nonlinear damping, which cannot be obtained by linear control. Another objective is to make full use of actuator power, rather than limiting it, so as to achieve a higher performance. Our approach is quite different from others in that the structure of the state feedback controller is not prescribed. Instead, it is derived as the result of stabilization. In this way, we succeed in obtaining a partial parameterization of nonlinear state feedback controllers. Then, it is shown that any linear observer can be applied to the nonlinear state feedback to realize nonlinear output feedback control.

Concretely speaking, a Lur'e–Postnikov type Lyapunov function is used to derive the stability condition, which is characterized by a PDMI about the feedback law. Then a class of solutions is derived analytically which has a nonlinear integral kernel as the free parameter. This leads to a partial parameterization of nonlinear controllers. The formula of solutions is explicit and easy to apply. This class is quite broad and the controller is allowed to have arbitrarily high gain in particular. So there exists a high possibility that the freedom in the nonlinear controllers can be used to optimize the control performance. Further, some examples are illustrated which show favorable system performance. Particularly, in the seek control design of a hard disk drive (HDD), comparison with linear control shows the superior performance of the proposed control method. Due to page limitations, only abbreviated proofs are shown. Detailed proofs can be found in Liu and Akasaka (2013).

Notations. For a matrix A, $A^\dagger$ denotes its Moore–Penrose inverse. $\ker A$ is the kernel space of A. Let Y_i be a square matrix, we use $\mathrm{diag}(Y_1, \ldots, Y_l)$ to denote the block diagonal matrix with diagonal block Y_i. Further, I_i denotes the ith dimensional identity matrix. $X_\perp$ is a row full rank matrix which has the largest rank among all matrices Y satisfying $YX = 0$. e_i is a vector whose ith entry is unit and the rest are all zeros. Also, $\mathrm{sgn}(\cdot)$ denotes the signum function.

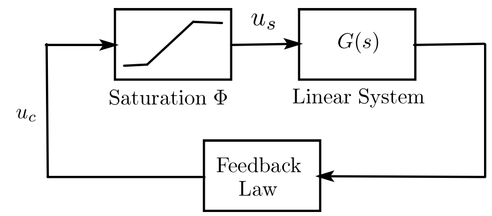

Fig. 1. Feedback control systems with saturation.

2. Problem statement

The feedback control system under consideration is illustrated in Fig. 1, which consists of a multi-input multi-output linear system $G(s)$, input saturation Φ and a feedback law. Since this paper focuses on the stabilization issue, the exogenous signal is omitted. The state equation of the system G is described by

$$G: \begin{cases} \dot{x} = Ax + Bu_s, & x \in \mathbb{R}^n,\ u_s \in \mathbb{R}^m \\ y = Cx, & y \in \mathbb{R}^p. \end{cases} \tag{1}$$

The control input $u_s = \Phi(u_c)$ is supplied through the actuator with saturation. Here, u_c denotes the control command and the saturation function $\Phi : \mathbb{R}^m \to \mathbb{R}^m$ is described as

$$\Phi(u_c) = \begin{bmatrix} \phi_1(u_{c_1}) & \cdots & \phi_m(u_{c_m}) \end{bmatrix}^T \tag{2}$$

where the ith element $\phi_i(u_{c_i})$ is any continuous function satisfying: (i) $\phi_i(u_{c_i}) = 0$ iff $u_{c_i} = 0$; and (ii) $u_{c_i}\phi_i(u_{c_i}) \geq 0$ (passivity). A typical example is the ideal saturation:

$$\phi_i(u_{c_i}) = \begin{cases} u_{c_i}, & |u_{c_i}| \leq m_i \\ \mathrm{sgn}(u_{c_i})m_i, & |u_{c_i}| > m_i \end{cases}$$

where $m_i > 0$ denotes the maximum magnitude of the ith control input.

Our goal is to design the input command u_c such that $u_s = \Phi(u_c)$ globally stabilizes the system (1). Both state and output feedback are considered. Since u_s is bounded due to the constraint of the actuator, not all linear systems are globally stabilizable. Thereafter, we make the following two assumptions on the system (1).

Assumption 1. (A, B, C) is controllable and observable.

Assumption 2. All eigenvalues of A are located in the closed left half plane. Further, the algebraic multiplicities of all Jordan blocks of zero eigenvalues are no greater than 2 and all pure imaginary eigenvalues are simple.

Unstable systems, such as the double integrator, are included in this class of systems. Assumption 1 is made for the output stabilization. The first part of Assumption 2 is necessary for the global stabilizability with a bounded input. The algebraic multiplicity conditions in Assumption 2 are made to ensure (i) the existence of a nontrivial matrix P satisfying

$$P \geq 0, \quad A^TP + PA \leq 0, \quad \ker A \subset \ker P; \tag{3}$$

and (ii) the radial unboundedness of the Lyapunov function:

$$V(x) = x^TPx + 2\sum_{i=1}^{m} \lambda_i \int_0^{s_i(x)} \phi_i(s)ds \quad (\lambda_i > 0). \tag{4}$$

Basically, the radial unboundedness of the Lyapunov function is required in order to guarantee the global stability. It can be proved that the algebraic multiplicity conditions made in Assumption 2 are necessary for this property (Liu & Akasaka, 2013).