Automatica 50 (2014) 247–254

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Event-triggered maximum likelihood state estimation☆

Dawei Shi[a,1], Tongwen Chen[a], Ling Shi[b]

[a] Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Alberta, Canada T6G 2V4

[b] Department of Electronic and Computer Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong

ARTICLE INFO

Article history:
Received 20 December 2012
Received in revised form
31 August 2013
Accepted 1 October 2013
Available online 4 November 2013

Keywords:
Wireless sensor networks
Riccati equations
Kalman filters
Dynamic programming
Event-triggered systems

ABSTRACT

The event-triggered state estimation problem for linear time-invariant systems is considered in the framework of Maximum Likelihood (ML) estimation in this paper. We show that the optimal estimate is parameterized by a special time-varying Riccati equation, and the computational complexity increases exponentially with respect to the time horizon. For ease in implementation, a one-step event-based ML estimation problem is further formulated and solved, and the solution behaves like a Kalman filter with intermittent observations. For the one-step problem, the calculation of upper and lower bounds of the communication rates from the process side is also briefly analyzed. An application example to sensorless event-based estimation of a DC motor system is presented and the benefits of the obtained one-step event-based estimator are demonstrated by comparative simulations.

1. Introduction

In wireless sensor networks, smart sensors and actuators are normally powered by batteries with limited capacity (Akyildiz, Su, Sankarasubramaniam, & Cayirci, 2002) and usually perform two types of tasks (Culler, Estrin, & Srivastava, 2004; Sveda & Vrba, 2003): simple calculation (including data acquisition) and data transmission via the wireless channel. The comparison between standard ZigBee chips designed according to IEEE 802.15.4 (2006) (e.g., CC2530 by Texas Instruments, 2011) and analog to digital converters (e.g., AD7988, 16-digit ADC from ANALOG DEVICES, 2012) indicates that the energy consumption of wireless transmission is at least one magnitude greater than that of data acquisition and basic calculation. Consequently, less communication between the sensor and the remote state estimator (or actuator) can significantly prolong the lifetime of the sensors. Event-based sensor data schedules provide an inspiring opportunity for reducing the sensor-to-estimator communications.

Pioneered by the work of Åström and Bernhardsson (2002) on Lebesgue sampling, event-based data scheduling for state estimation has received considerable attention during the last few years. The optimal event-based finite-horizon sensor transmission scheduling problems were studied in Imer and Basar (2005), and Rabi, Moustakides, and Baras (2006) for continuous-time and discrete-time scalar linear systems, respectively. The results were extended to vector linear systems in Li, Lemmon, and Wang (2010) by relaxing the zero mean initial conditions and considering measurement noises. The tradeoff between the performance and the average sampling period was analyzed in Li and Lemmon (2011), and a sup-optimal event-triggering scheme with a guaranteed least average sampling period was proposed. Adaptive sampling for state estimation of continuous-time linear systems was considered in Rabi, Moustakides, and Baras (2012). Shi, Johansson, and Qiu (2011) proposed a hybrid sensor data scheduling method by combining time and event-based methods with reduced computational complexity. In Weimer, Araújo, and Johansson (2012), a distributed event-triggered estimation problem was considered and a global event-triggered communication policy for state estimation was proposed by minimizing a weighted function of network energy consumption and communication cost while considering estimation performance constraints. The joint design of event-trigger and estimator for first-order stochastic systems with arbitrary distributions was considered in Molin and Hirche (2012), where a game-theoretic framework was utilized to analyze the optimal trade-off between the mean squared estimation error and the expected transmission rate.

In addition to the scheduling issues, another important problem is to find the optimal estimate for a specified event-triggering scheme, which provides additional information to the estimator

☆ This work was supported by Natural Sciences and Engineering Research Council of Canada; in addition, the work by L. Shi was supported by grant DAG12EG09S. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor James Lam under the direction of Editor Ian R. Petersen.

E-mail addresses: dshi@ualberta.ca (D. Shi), tchen@ualberta.ca (T. Chen), eesling@ust.hk (L. Shi).

[1] Tel.: +1 780 886 5780; fax: +1 780 492 1811.

0005-1098/$ – see front matter
http://dx.doi.org/10.1016/j.automatica.2013.10.005

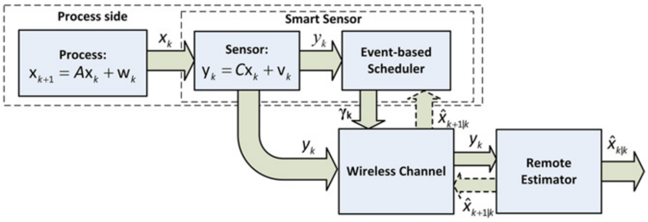

Fig. 1. Block diagram of the overall system. Note that for the proposed one-step event-triggered ML estimator, the feedback communication from the remote estimator to the smart sensor (see the dotted arrow) is only required when an event occurs at the smart sensor.

even when no measurement is transmitted from the sensor. In Wu, Jia, Johansson, and Shi (2013), the Minimum Mean Squared Error (MMSE) estimator was derived based on the Gaussian assumption of the prediction error, and the tradeoff between the sensor-to-estimator communication rate and the performance was analytically characterized. In Sijs and Lazar (2012), a general description of event sampling was proposed, and a state estimator with a hybrid update was derived using a sum of Gaussians approach to reduce the computational complexity. Trimpe and D'Andrea (2012) considered the variance-based event-triggering conditions, and the convergence of the resultant variance iterations to a periodic solution was proved. For linear Gaussian systems with periodic sensor measurements, the MMSE estimate, namely, the Kalman filter, coincides with the Maximum Likelihood (ML) estimate (Rauch, Striebel, & Tung, 1965). However, this equivalence no longer holds when the sensor measurements are updated according to an event-triggered scheme, due to the non-Gaussianity of the conditional Probability Distribution Functions (PDFs).

In this paper, the event-based state estimation problem is considered under the maximum-likelihood estimation framework. We study the remote state estimation of a process based on the measurements taken by a battery-powered smart-sensor on the process side, the output of which is transmitted to the remote estimator through a wireless channel. Foreshadowed by the discussions above, we assume that wireless transmission consumes more energy than basic calculation, and thus an event-based data-scheduler is proposed on the process side to prolong the battery life (utilizing the limited calculation capacity of the smart sensor). The main contributions of this paper are two folds:

(1) The structure of the event-based ML state estimator is provided. We show that the optimal estimate is parameterized by a special time-varying Riccati equation, and the computational complexity increases exponentially with the time horizon. Note that the solution to the Riccati equation is not necessarily the covariance matrix of the estimation error for event-based ML state estimation problems, due to event-based data updating.

(2) For ease in implementation of the event-based ML estimator, a one-step event-based ML estimation problem is formulated, and its solution is shown to behave like the Kalman filter with intermittent observations (Sinopoli et al., 2004) and only requires feedback communication when an event occurs at the smart sensor. This is different from the results in Wu et al. (2013), where feedback communication is always needed. Also, discussions on the communication rates are provided from the process side.

The rest of the paper is organized as follows. Section 2 provides the system description and problem formulation. The structure of the solution to the event-based ML state estimation problem is derived in Section 3, where the implementation issues are also discussed. In Section 4, the one-step event-based ML estimation problem is solved and the communication rate is briefly analyzed. Section 5 presents a numerical example to illustrate the efficiency of the proposed results, followed by some concluding remarks in Section 6.

Notations: $\mathbb{N}$ and $\mathbb{N}^+$ denote the sets of nonnegative and positive integers, respectively. For $a,\ b \in \mathbb{N}$ and $a \le b$, $u_{a:b}$ denotes $\{u(a), u(a+1), \ldots, u(b)\}$. $\mathbb{R}$ denotes the set of real numbers. For $m,\ n \in \mathbb{N}^+$, $\mathbb{R}^{m\times n}$ denotes the set of m by n real-valued matrices, whereas $\mathbb{R}^m$ is short for $\mathbb{R}^{m\times 1}$. For $Z \in \mathbb{R}^{m\times n}$, $Z^\top$ denotes the transpose of Z, whereas $Z^{-\top}$ denotes $(Z^\top)^{-1}$ if Z is square and nonsingular. For a random variable x, $\mathbf{E}(\mathrm{x})$ denotes its expectation, and x denotes its realization.

2. Problem formulation

Consider the system in Fig. 1. The process is Linear Time-Invariant (LTI) and evolves in discrete time driven by white noise:

$$\mathrm{x}_{k+1} = A\mathrm{x}_k + \mathrm{w}_k, \tag{1}$$

where $\mathrm{x}_k \in \mathbb{R}^n$ is the state, and $\mathrm{w}_k \in \mathbb{R}^n$ is the noise input, which is zero-mean Gaussian with covariance $Q > 0$.

The initial state x_0 is Gaussian with $\mathbf{E}(\mathrm{x}_0) = \mu_0$ and covariance $P_0 > 0$. Assume that A is nonsingular. Note that this assumption is not restrictive as (1) is typically a model that comes from discretizing a stochastic differential equation $\mathrm{d}x = A_1 x\mathrm{d}t + B_1\mathrm{d}w$, in which case $A = e^{A_1 h}$, for a sampling period h, is clearly invertible. The state information is measured by a battery-powered smart sensor, which communicates with a remote state estimator through a wireless channel, and the measurement equation is

$$\mathrm{y}_k = C\mathrm{x}_k + \mathrm{v}_k, \tag{2}$$

where $\mathrm{v}_k \in \mathbb{R}^m$ is zero-mean Gaussian with covariance $R > 0$. In addition, x_0, w_k and v_k are uncorrelated with each other. We assume that (C, A) is detectable. For consideration of the limited sensor battery capacity and the communication cost, an event-based data scheduler is integrated in the sensor. At each time instant k, the measurement information y_k is sent directly to the event-based scheduler; the estimator provides a prediction $\hat{x}_{k|k-1}$ of the current state x_k and sends the prediction $\hat{x}_{k|k-1}$ to the scheduler via the wireless channel. Based on y_k and $\hat{x}_{k|k-1}$, the scheduler computes γ_k according to the following event-triggered condition:

$$\gamma_k = \begin{cases} 0, & \text{if } \|y_k - C\hat{x}_{k|k-1}\|_\infty \le \delta \\ 1, & \text{otherwise} \end{cases} \tag{3}$$

and decides whether to allow a data transmission, where δ is a tuning parameter that determines the sensitivity of the event-based scheduler. Only when $\gamma_k = 1$, the sensor transmits y_k to the estimator. As a result, if $\gamma_k = 1$, the estimator knows the exact value of y_k; otherwise it only knows that the value of y_k lies in a known region. The ultimate goal of the estimator is to provide an estimate $\hat{x}_{k|k}$ of x_k based on the known information. Notice that this type of feedback communication strategy is not energy-saving itself and an alternative strategy is to include a copy of the estimator in the scheduler, which instead adds to the computational burden of the scheduler. We will show that the obtained result in this paper in fact does not require the feedback communication except when an event occurs, and only a simple prediction step is needed for the scheduler during a non-event time instant.

In this paper, the first objective is to determine, at time k, the optimal estimate $\hat{x}_{k|k}$ of x_k that maximizes the joint probability distribution function of $\mathrm{x}_{0:k}$ and $\mathrm{y}_{1:k}$:

$$f_{\mathrm{x}_{0:k},\mathrm{y}_{1:k}}(\hat{x}_{0|0}, x_1, \ldots, x_k, \hat{y}_1, \ldots, \hat{y}_k) \tag{4}$$

where $x_{0:k}$ and $\hat{y}_{1:k}$ are the optimization parameters. If $\gamma_t = 1$, $\hat{y}_t = y_t$; otherwise the value of $\hat{y}_t$ lies in $[\underline{y}_t,\ \bar{y}_t]$ at time instant t, where

$$\underline{y}_t = C\hat{x}_{t|t-1} - \delta\mathbf{1}_m,$$
$$\bar{y}_t = C\hat{x}_{t|t-1} + \delta\mathbf{1}_m,$$