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On iterative learning algorithms for the formation control of nonlinear multi-agent systems☆

Deyuan Meng[a,1], Yingmin Jia[a], Junping Du[b], Jun Zhang[c]

[a] The Seventh Research Division and the Department of Systems and Control, Beihang University (BUAA), Beijing 100191, PR China
[b] Beijing Key Laboratory of Intelligent Telecommunications Software and Multimedia, School of Computer Science and Technology, Beijing University of Posts and Telecommunications, Beijing 100876, PR China
[c] School of Electronic and Information Engineering, Beihang University (BUAA), Beijing 100191, PR China

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ABSTRACT

This paper deals with formation control problems for multi-agent systems with nonlinear dynamics and switching network topologies. Using the nearest neighbor knowledge, a distributed algorithm is constructed by employing the iterative learning control approach. Sufficient conditions are given to obtain the desired relative formations of agents, which benefits from the strict positiveness of products of stochastic matrices. It is shown that the derived results can effectively work, although the network topologies dynamically change along both time and iteration axes and the corresponding directed graphs may not have spanning trees. Such result is also illustrated via numerical simulations.

1. Introduction

Multi-agent systems usually consist of a group of agents cooperating to complete certain tasks for the group, and their coordination control has generated considerable research interest (see, e.g., Jadbabaie, Lin, & Morse, 2003; Moreau, 2005; Nedic, Olshevsky, Ozdaglar, & Tsitsiklis, 2009; Olfati-Saber & Murray, 2004; Olshevsky & Tsitsiklis, 2009; Ren & Beard, 2005; Schenato & Fiorentin, 2011). These results contribute greatly to achieving coordination control objectives as time goes to infinity, especially in the case when the time-varying communication graphs are explored to describe the switching network topologies of multi-agent systems. In addition, there exists a class of practical coordination tasks that are required to be performed as accurately as possible over a finite time horizon, such as the formation of mobile robots in Chen and Jia (2010) and trajectory-keeping of flying satellites in Ahn, Moore, and Chen (2010). This class of coordination tasks may not be well accomplished by traditional or conventional multi-agent approaches in the literature, which motivates some recent studies on applying iterative learning control (ILC) approaches to deal with them, such as relative formation (Ahn & Chen, 2009; Liu & Jia, 2012; Xu, Zhang, & Yang, 2011) and consensus tracking (Meng & Jia, 2012; Meng, Jia, Du, & Yu, 2012, 2013). It has been shown that the coordination tasks with the high precision requirement can be well carried out for multi-agent systems through an ILC process. This is also the motivation of the present paper that is devoted to the combined studies of ILC and multi-agent approaches on the formation control issues for multiple agents with nonlinear dynamics and switching topologies dynamically changing along both time and iteration axes.

It is worth pointing out that the existing results of multi-agent ILC require the fixed network topology except that of Liu and Jia (2012) which, however, requires all the switching graphs to have spanning trees at all time and all iterations. In contrast to this, our paper aims to relax such requirements on the formation learning control of nonlinear multi-agent systems with switching topologies. We propose an ILC algorithm using the nearest neighbor rule and develop properties of stochastic matrices to give a relaxed convergence condition. This condition can allow even none of the switching graphs to have spanning trees and ensure the learning

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E-mail addresses: dymeng23@126.com (D. Meng), ymjia@buaa.edu.cn (Y. Jia), junpingdu@126.com (J. Du), buaazhangjun@vip.sina.com (J. Zhang).

[1] Tel.: +86 10 8233 8683; fax: +86 10 8231 7933.

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process with an exponential convergence speed, which takes advantage of the connected property of switching graphs (see, e.g., Jadbabaie et al., 2003; Moreau, 2005; Nedic et al., 2009; Olfati-Saber & Murray, 2004; Schenato & Fiorentin, 2011). We also illustrate this result by numerical simulations.

Notations: $\mathscr{I}_n = \{1, 2, \ldots, n\}$, $\mathbb{Z}_N = \{0, 1, \ldots, N\}$, $1_n = [1, 1, \ldots, 1]^{\mathrm{T}} \in \mathbb{R}^n$, $\|A\|_\infty$ is the maximum row sum norm (respectively, the l_∞ norm) of a matrix (respectively, a vector) A, $A > 0$ (respectively, $A \geq 0$) if its elements are all positive (respectively, nonnegative), and $A \circ B$ is the Hadamard product of A and B. A nonnegative matrix $A \in \mathbb{R}^{n\times n}$ is said to be stochastic if A satisfies $A1_n = 1_n$.

Preliminaries in graph theory: let $\mathscr{G} = (\mathscr{V}(\mathscr{G}), \mathscr{E}(\mathscr{G}))$ be an nth order directed graph, where $\mathscr{V}(\mathscr{G}) \triangleq \mathscr{V} = \{v_i : i \in \mathscr{I}_n\}$ is the vertex set, and $\mathscr{E}(\mathscr{G}) \triangleq \mathscr{E} \subseteq \{(v_i, v_j) : v_i, v_j \in \mathscr{V}\}$ is the edge set. An edge (v_j, v_i) corresponds to a channel where information flows from v_j to v_i. The index set of the neighbors of v_i is denoted by $\mathscr{N}_i = \{j : (v_j, v_i) \in \mathscr{E}\}$. A path in $\mathscr{G}$ is a finite sequence $v_{i_1}, v_{i_2}, \ldots, v_{i_j}$ of vertices such that $(v_{i_{l+1}}, v_{i_l}) \in \mathscr{E}$ for $l = 1, 2, \ldots, j-1$. If there exists a special vertex that can be connected to all the other vertices through paths, $\mathscr{G}$ is said to have a spanning tree. If there exists a path between any distinct pair of vertices, $\mathscr{G}$ is said to be strongly connected. For $\mathscr{G}_1 = (\mathscr{V}, \mathscr{E}_1)$ and $\mathscr{G}_2 = (\mathscr{V}, \mathscr{E}_2)$, $\mathscr{G} = \mathscr{G}_1 \bigcup \mathscr{G}_2$ is called the union of $\mathscr{G}_1$ and $\mathscr{G}_2$, where $\mathscr{E} = \mathscr{E}_1 \bigcup \mathscr{E}_2$. A nonnegative weighted adjacency matrix $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ associated with $\mathscr{G}$ is defined to model the information exchange between agents, where $a_{ij} > 0 \Leftrightarrow (v_j, v_i) \in \mathscr{E}$ and $a_{ij} = 0$ otherwise. Here, we assume $a_{ii} = 0$ for $i \in \mathscr{I}_n$. The Laplacian matrix of $\mathscr{G}$ is defined as $\mathscr{L}_{\mathscr{A}} = \Delta_{\mathscr{A}} - \mathscr{A}$, where $\Delta_{\mathscr{A}} = \mathrm{diag}\left\{\sum_{j\in\mathscr{N}_1} a_{1j}, \sum_{j\in\mathscr{N}_2} a_{2j}, \ldots, \sum_{j\in\mathscr{N}_n} a_{nj}\right\}$. The extended directed graph of $\mathscr{G}$ with all n self-loops is denoted by $\mathscr{G}^{sl} = (\mathscr{V}(\mathscr{G}^{sl}), \mathscr{E}(\mathscr{G}^{sl}))$, where $\mathscr{V}(\mathscr{G}^{sl}) = \mathscr{V}$ and $\mathscr{E}(\mathscr{G}^{sl}) = \mathscr{E} \bigcup \{(i, i), \forall i \in \mathscr{I}_n\}$. A stochastic matrix $S = [S_{ij}] \in \mathbb{R}^{n\times n}$ is said to be associated with $\mathscr{G}^{sl}$ if $S_{ij} > 0 \Leftrightarrow (v_j, v_i) \in \mathscr{E}(\mathscr{G}^{sl})$ for all $i, j \in \mathscr{I}_n$. Clearly, a stochastic matrix associated with $\mathscr{G}^{sl}$ must have positive diagonal elements.

2. Problem description

We consider networked systems that consist of n mobile agents labeled 1 through n, where all agents share a common state space $\mathbb{R}$. Let t be the time variable taking values from a finite interval $\mathbb{Z}_N$. The ith agent is considered to have the following dynamics over $t \in \mathbb{Z}_N$:

$$x_i(t+1) = f_i(x_i(t)) + u_i(t), \quad i \in \mathscr{I}_n \tag{1}$$

where $x_i(t)$ is the state, $u_i(t)$ is the protocol, and $f_i(\cdot)$ is a continuously differentiable nonlinear function with range in $\mathbb{R}$ and its differential is uniformly bounded, i.e., $|df_i(x)/dx| \leq b_{f_i}$ holds with a finite bound $0 \leq b_{f_i} < \infty$ for $x \in \mathbb{R}$.

Let $k \in \mathbb{Z}_+$ denote the iteration index (independent of t). At the kth iteration, the state and protocol of (1) are denoted by $x_{i,k}(t)$ and $u_{i,k}(t)$, respectively. Then by taking the iterative process into account, the dynamics of the ith agent given in (1) are rewritten as

$$x_{i,k}(t+1) = f_i\left(x_{i,k}(t)\right) + u_{i,k}(t), \quad i \in \mathscr{I}_n \tag{2}$$

where the state $x_{i,k}(t)$ satisfies the initial resetting condition: $x_{i,k}(0) = x_{i0}$ for $k \in \mathbb{Z}_+$. For the agents given by (2), we consider their communication associated with a directed graph dynamically changing with respect to both t and k, which is denoted by $\mathscr{G}_k(t)$. Let every agent be regarded as a vertex in $\mathscr{G}_k(t)$. Consequently, every edge $(v_j, v_i) \in \mathscr{E}(\mathscr{G}_k(t))$ corresponds to an available information channel from the agent v_j to the agent v_i at the time step t and iteration k. The weighted adjacency matrix associated with $\mathscr{G}_k(t)$ depends on both t and k, which is denoted by $\mathscr{A}_k(t) = \left[a_{ij,k}(t)\right]$. Thus, the extended directed graph of $\mathscr{G}_k(t)$ with all self-loops is denoted by $\mathscr{G}_k^{sl}(t)$, and the Laplacian matrix and the neighbor index set of the ith agent associated with $\mathscr{A}_k(t)$ are denoted by $\mathscr{L}_{\mathscr{A}_k(t)}$ and $\mathscr{N}_{i,k}(t)$, respectively. In addition, let $\overline{\mathscr{G}}_s = \{\mathscr{G}_{s_1}, \mathscr{G}_{s_2}, \ldots, \mathscr{G}_{s_v}\}$ be the set of all directed graphs that are defined for the agents, i.e., $\mathscr{G}_k(t) \in \overline{\mathscr{G}}_s$ for all t and k.

For two agents v_i and v_j, we denote $x_{ij,k}(t) = x_{i,k}(t) - x_{j,k}(t)$ as the relative formation between them. Let $d_{ij}(t)$ be the desired relative formation between the two agents v_i and v_j over $t \in \mathbb{Z}_N$. The problem considered in this paper is to make the multi-agent system (2) achieve the desired relative formation, i.e., for $\forall i, j \in \mathscr{I}_n$,

$$\lim_{k\to\infty} x_{ij,k}(t) = d_{ij}(t), \quad t \in [1, N]. \tag{3}$$

Note that $d_{ij}(t)$ can be given in the form of $d_{ij}(t) = d_i(t) - d_j(t)$, where $d_i(t)$ is the desired state deviation trajectory of the agent v_i to an unknown common reference or leader agent for the multi-agent system (see, e.g., Liu & Jia, 2012; Meng et al., 2012, 2013). To achieve the objective (3) based on the nearest neighbor information, we use the iterative rules to construct a formation algorithm as

$$u_{i,k+1}(t) = u_{i,k}(t) + \sum_{j\in\mathscr{N}_{i,k}(t)} \phi_{ij,k}(t) a_{ij,k}(t) \times \left[d_{ij}(t+1) - x_{ij,k}(t+1)\right], \quad i \in \mathscr{I}_n \tag{4}$$

where $u_{i,0}(t)$ is the initial input, and $\phi_{ij,k}(t) \geq 0$ is the nonnegative learning gain. Let $\Phi_k(t) = \left[\phi_{ij,k}(t)\right] \in \mathbb{R}^{n\times n}$, and thus $\Phi_k(t) \geq 0$. As shown in Meng and Jia (2012) and Meng et al. (2012), such a gain matrix is always selected such that

$$\phi_{ij,k}(t) \begin{cases} > 0, & j \in \mathscr{N}_{i,k}(t) \\ = 0, & \text{otherwise} \end{cases}$$

which guarantees that $\Phi_k(t) \circ \mathscr{A}_k(t) \geq 0$ can be an adjacency matrix associated with $\mathscr{G}_k(t)$. Since $\mathscr{G}_k(t) \in \overline{\mathscr{G}}_s$, it is natural to consider that $\Phi_k(t)$ takes its values from a set with finite elements in accordance with $\mathscr{G}_k(t)$, i.e., $\Phi_k(t) \in \overline{\Phi}_s \triangleq \{\Phi_{s_1}, \Phi_{s_2}, \ldots, \Phi_{s_v}\}$.

Remark 1. It is worth pointing out that the construction of (4) takes into account the relative degree of the agents' plant. From (1) or (2), it can be found that we consider the plant with relative degree one for the agents (i.e., there is one step of time delay in the state of every agent, in order to have the corresponding control input appearing). This fact motivates us to apply the state information at the time step $t+1$ (i.e., $x_{ij,k}(t+1)$) to develop the updated protocol at the time step t (i.e., $u_{i,k+1}(t)$) in the formation learning algorithm (4). But, it is worth noting that the information $x_{ij,k}(t+1)$ is from the iteration k and, thus, available when computing the protocol $u_{i,k+1}(t)$ for the agents to operate at the next iteration $k+1$. Actually, this reflects one of the typical design methods for ILC of the discrete-time plants with relative degree one (see, e.g., the ILC survey Ahn, Chen, & Moore, 2007 for more details).

3. Main results

Let $e_{i,k}(t) = d_i(t) - x_{i,k}(t)$ denote the state deviation error of v_i. It can be seen that the objective (3) holds if there exists a function $c(t) \in \mathbb{R}$ such that $\lim_{k\to\infty} e_k(t+1) = c(t)1_n$ for $t \in \mathbb{Z}_{N-1}$, where $e_k(t) = \left[e_{1,k}(t), e_{2,k}(t), \ldots, e_{n,k}(t)\right]^{\mathrm{T}}$. We will thus consider how to deal with this convergence problem instead in our following discussions. Toward this end, we introduce an auxiliary variable as $\eta_{i,k}(t) = x_{i,k+1}(t) - x_{i,k}(t)$, and then denote $\eta_k(t) = \left[\eta_{1,k}(t), \eta_{2,k}(t), \ldots, \eta_{n,k}(t)\right]^{\mathrm{T}}$.

By combining (2) and (4), we can derive

$$\begin{aligned} \eta_{i,k}(t+1) &= \left[f_i\left(x_{i,k+1}(t)\right) - f_i\left(x_{i,k}(t)\right)\right] + \left[u_{i,k+1}(t) - u_{i,k}(t)\right] \\ &= \left[f_i\left(x_{i,k+1}(t)\right) - f_i\left(x_{i,k}(t)\right)\right] + \sum_{j\in\mathscr{N}_{i,k}(t)} \phi_{ij,k}(t) \\ &\quad \times a_{ij,k}(t)\left[e_{i,k}(t+1) - e_{j,k}(t+1)\right]. \end{aligned} \tag{5}$$