Automatica 48 (2012) 2560–2568

Contents lists available at SciVerse ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Decentralized output-feedback control of large-scale nonlinear systems with sensor noise ☆

Tengfei Liu [a,1], Zhong-Ping Jiang [a], David J. Hill [b]

[a] Department of Electrical and Computer Engineering, Polytechnic Institute of New York University, Six Metrotech Center, Brooklyn, NY 11201, USA

[b] School of Electrical and Information Engineering, The University of Sydney, NSW 2006, Australia

ARTICLE INFO

Article history:
Received 6 November 2010
Received in revised form
25 April 2012
Accepted 1 May 2012
Available online 15 July 2012

Keywords:
Decentralized control
Large-scale nonlinear systems
Output feedback
Sensor noise
Input-to-state stability (ISS)
Small-gain

ABSTRACT

This paper presents a new tool for decentralized output-feedback control design of large-scale nonlinear systems in the presence of non-smooth sensor noise. Through a recursive control design approach, the closed-loop decentralized system is transformed into a network of input-to-state stable (ISS) systems and the influences of the sensor noise are represented by ISS gains. The decentralized control objective is achieved by applying the cyclic-small-gain theorem to the closed-loop decentralized system. Moreover, the outputs of the closed-loop decentralized system can be driven arbitrarily close to the levels of their corresponding sensor noise.

1. Introduction

Decentralized control problems arise from various engineering applications, such as power systems, transportation networks, water systems, chemical engineering and telecommunication networks. Significant contributions have been made to the development of decentralized nonlinear control theory; see e.g., Arslan and Başar (2003), Jain and Khorrami (1997), Jiang, Repperger, and Hill (2001), Shi and Singh (1993), Šiljak (1991), Spooner and Passino (1996), Stanković, Stipanović, and Šiljak (2007), Wen (1994), Wen, Zhou, and Wang (2009), Xie and Xie (1998) and Ye and Huang (2003) to name only a few.

In this paper, we study for the first time the decentralized output-feedback control problem of large-scale nonlinear systems in the presence of non-smooth sensor noise. Beyond the robustness of the decentralized controllers with respect to sensor noise, we are also concerned about reducing the influence of the sensor noise on control performance. This problem is closely related to the nonlinear measurement feedback control problem. In Freeman and Kokotović (1996), the robust controller was designed with "flattened" Lyapunov functions following a nontrivial modification of the backstepping methodology (Krstić, Kanellakopoulos, & Kokotović, 1995). The closed-loop system is made input-to-state stable (ISS) with respect to the measurement errors. See Sontag (2007) for a tutorial of ISS. The "flattened" Lyapunov function appears to be a powerful tool for dealing with the non-smoothness of the measurement errors. However, in that result, the influence of the measurement errors grows with the order of the system. Fah (1999) studied the input-to-state stabilization of first-order nonlinear systems with measurement error. In Ledyaev and Sontag (1999), the conditions under which a system can be stabilized insensitively to small measurement errors were given. Jiang, Mareels, and Hill (1999) studied nonlinear systems composed of two subsystems, one is input-to-state stable and the other one is input-to-state stabilizable with respect to the measurement noise.

The problem studied in our paper is technically challenging due to two reasons: (a) The non-smoothness and even discontinuity of the sensor noise and the presence of dynamic uncertainty prevent the standard methods such as backstepping from being directly applicable; (b) There has not been an effective method

☆ This research was supported partly under the Australian Research Councils Discovery funding scheme (project number: FF0455875), partly by a seed grant from NYU and POLY, and by NSF grants DMS-0906659 and ECS-1101401. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Murat Arcak under the direction of Editor Andrew R. Teel.

E-mail addresses: tliu@poly.edu (T. Liu), zjiang@poly.edu (Z.-P. Jiang), davidh@ee.usyd.edu.au (D.J. Hill).

[1] Tel.: +1 718 260 3687; fax: +1 718 260 3906.

0005-1098/$ – see front matter
doi:10.1016/j.automatica.2012.06.054

to evaluate and reduce the influence of the sensor noise on control performance for large-scale high-order nonlinear systems. From the viewpoint of sensor noise, when reduced to the single (centralized) system, our problem appears to be more general than the previously studied problems.

In this paper, we propose a new design tool to solve the aforementioned decentralized output-feedback control problem with non-smooth sensor noise. We will design ISS induced decentralized reduced-order observers to estimate the unmeasured states, and to transform the decentralized output-feedback control problem into a decentralized state-feedback control problem. To deal with the non-smooth measurement noise and dynamic uncertainties, decentralized control laws will be designed in a recursive manner to transform the closed-loop decentralized system into an interconnection of ISS subsystems, and in particular, the influences of the sensor noise will be represented by ISS gains.

Based on the ISS small-gain theorem in Jiang, Teel, and Praly (1994) and Jiang, Mareels, and Wang (1996), the recently developed cyclic-small-gain theorem in Teel (2003), Jiang and Wang (2008) and Liu, Hill, and Jiang (2011) is a new tool for ISS designs of network systems. In our paper, we will employ the cyclic-small-gain theorem to guarantee the ISS of the closed-loop decentralized system, and moreover, to effectively evaluate the influence (represented by ISS gains) of the sensor noise by constructing ISS-Lyapunov functions. With the cyclic-small-gain based design, the outputs of the closed-loop decentralized system are driven arbitrarily close to the levels of their corresponding sensor noise. For other recent developments of small-gain theorems for large-scale nonlinear systems, the reader should consult (Dashkovskiy, Rüffer, & Wirth, 2007, 2010; Karafyllis & Jiang, 2011).

To make the paper self-contained, recall that $\mathbb{R}_+$ represents the set of nonnegative real numbers. A function $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ is positive definite if $\gamma(s) > 0$ for all $s > 0$ and $\gamma(0) = 0$. $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ is a $\mathcal{K}$ function (denoted by $\gamma \in \mathcal{K}$) if it is continuous, strictly increasing and $\gamma(0) = 0$; it is a $\mathcal{K}_\infty$ function (denoted by $\gamma \in \mathcal{K}_\infty$) if it is a $\mathcal{K}$ function and also satisfies $\gamma(s) \to \infty$ as $s \to \infty$. Id represents the identity function. I_n represents the identity matrix of size n. $\lambda_{\max}(P)$ means the largest eigenvalue of a real and symmetric matrix P.

2. Problem formulation

Consider a class of large-scale systems composed of N subsystems in disturbed output-feedback form:

$$\dot{z}_i = \Delta_{i0}(z_i, y_i, w_i) \tag{1}$$

$$\dot{x}_{ij} = x_{i(j+1)} + \Delta_{ij}(y_i, z_i, w_i), \quad 1 \le j \le n_i \tag{2}$$

$$y_i = x_{i1} \tag{3}$$

$$x_{i(n_i+1)} \stackrel{\text{def}}{=} u_i \tag{4}$$

$$w_i = [y_1, \dots, y_{i-1}, y_{i+1}, \dots, y_N, d_i^{eT}]^T \tag{5}$$

$$y_i^m = y_i + d_i^m \tag{6}$$

where, for each $1 \le i \le N$, $[z_i^T, x_{i1}, \dots, x_{in_i}]^T$ with $z_i \in \mathbb{R}^{n_{z_i}}$ and $x_{ij} \in \mathbb{R}\,(j = 1, \dots, n_i)$ is the state, $u_i \in \mathbb{R}$ is the control input, $y_i \in \mathbb{R}$ is the output, z_i and $[x_{i2}, \dots, x_{in_i}]^T$ are the unmeasured portions of the state, $y_i^m \in \mathbb{R}$ is the measurement of the output, $d_i^e \in \mathbb{R}^{n_{di}}$ and $d_i^m \in \mathbb{R}$ are external and measurement errors, respectively, and Δ_{ij}'s $(1 \le j \le n_i)$ are uncertain locally Lipschitz functions.

Remark 1. The disturbed output-feedback form considered in (1)–(6) is a variant of the measurement error-free system considered in Jiang et al. (2001) and occurs in some mechanical systems, e.g., the interconnected system of cart-inverted double pendulum in Shi and Singh (1993) and Šiljak (1991). Differently from centralized systems, the input/output feedback linearization cannot be implemented in the decentralized system due to the dependence of the Δ_{ij}'s on w_i. In particular, the z_i-subsystem is referred to as the nonlinear inverse dynamics, which forms nonlinear dynamical interactions between the subsystems of the decentralized system.

The following assumptions are made on system (1)–(6).

Assumption 1. For each Δ_{ij} $(1 \le i \le N,\ 1 \le j \le n_i)$, there exists a known $\psi_{\Delta_{ij}} \in \mathcal{K}_\infty$ such that for all z_i, y_i, w_i,

$$|\Delta_{ij}(z_i, y_i, w_i)| \le \psi_{\Delta_{ij}}(|[z_i^T, y_i, w_i^T]^T|). \tag{7}$$

Assumption 2. For each $1 \le i \le N$, there exist $\bar{d}_i^e, \bar{d}_i^m \ge 0$, such that

$$|d_i^e(t)| \le \bar{d}_i^e, \quad \forall t \ge 0; \qquad |d_i^m(t)| \le \bar{d}_i^m, \quad \forall t \ge 0. \tag{8}$$

Assumption 3. Each z_i-subsystem in (1) with y_i and w_i as the inputs admits a smooth ISS-Lyapunov function V_{z_i}, satisfying the following properties:

1. There exist $\underline{\alpha}_{z_i}, \overline{\alpha}_{z_i} \in \mathcal{K}_\infty$ such that for all z_i,

$$\underline{\alpha}_{z_i}(|z_i|) \le V_{z_i}(z_i) \le \overline{\alpha}_{z_i}(|z_i|). \tag{9}$$

2. There exist $\chi_{z_i}^{y_i}, \chi_{z_i}^{w_i} \in \mathcal{K}_\infty$ and α_{z_i} continuous and positive definite such that for all z_i, y_i, w_i,

$$V_{z_i}(z_i) \ge \max\{\chi_{z_i}^{y_i}(|y_i|), \chi_{z_i}^{w_i}(|w_i|)\}$$
$$\Rightarrow \nabla V_{z_i}(z_i)\Delta_{i0}(z_i, y_i, w_i) \le -\alpha_{z_i}(V_{z_i}(z_i)). \tag{10}$$

The objective of this paper is to design a class of decentralized controllers for the large-scale system composed of (1)–(6), such that the outputs y_i's $(1 \le i \le N)$ are steered to within some small neighborhood of the origin.

3. Main result

In this section, we first present a modified gain assignment lemma as a basic design ingredient. Then, we design reduced-order observers to transform the decentralized output-feedback control problem into a decentralized state-feedback control problem. In Section 3.3, we develop a small-gain control design approach based on worst-case virtual control laws to overcome the problems caused by the nonsmoothness of the sensor noise and to transform the closed-loop decentralized system into a network of ISS systems. In Sections 3.4 and 3.5, we employ the Lyapunov-based ISS cyclic-small-gain theorem (see Appendix B) to guarantee the ISS of the closed-loop decentralized system and to fine tune the control parameters to effectively attenuate the influence of the sensor noise.

3.1. Gain assignment lemma

The gain assignment technique has been shown extremely useful in small-gain based nonlinear control designs in Jiang and Mareels (1997), Jiang et al. (1999), Jiang et al. (1994) and Praly and Wang (1996). In this subsection, we present a modified gain assignment lemma.

Consider the following first-order system:

$$\dot{\eta} = \phi(\eta, \omega_1, \dots, \omega_n) + \mu \tag{11}$$

$$\eta^m = \eta + \omega_{n+1} + \mathrm{sgn}(\eta)|\omega_{n+2}| \tag{12}$$

where $\eta \in \mathbb{R}$ is the state, $\mu \in \mathbb{R}$ is the control input, $\omega_1, \dots, \omega_{n+2} \in \mathbb{R}$ represent the external disturbance inputs, sgn denotes the standard sign function, $\eta^m \in \mathbb{R}$ represents the measurement of