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Brief paper

A Bayesian solution to the multiple composite hypothesis testing for fault diagnosis in dynamic systems[☆]

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ABSTRACT

This paper is concerned with model-based isolation and estimation of additive faults in discrete-time linear Gaussian systems. The isolation problem is stated as a multiple composite hypothesis testing on the innovation sequence of the Kalman filter (KF) that considers the system operating under fault-free conditions. Fault estimation is carried out, after isolating a fault mode, by using the *Maximum a Posteriori* (MAP) criterion. An explicit solution is presented for both fault isolation and estimation when the parameters of the fault modes are assumed to be realizations of specific random variables (RV).

1. Introduction

The Generalized Likelihood Ratio Test (GLRT) proposed by Willsky and Jones (1976) will be adopted here as the starting point for the development of a new method for fault diagnosis. It consists of a statistical test performed on the innovation sequence of the Kalman filter (KF) that assumes there exists no fault in the system. This method does not assume any prior knowledge about the fault parameters.

This paper is concerned with diagnosis (isolation and estimation) of additive faults in discrete-time linear systems excited by Gaussian noise. The innovation sequence of the KF is used as a residual signal. The isolation problem is stated as a multiple composite hypothesis testing, for which an approximated Bayesian solution is proposed. After isolating a fault mode, the fault parameters are estimated by means of the *Maximum a Posteriori* (MAP) criterion. Unlike the approaches available in the literature, the proposed method does not require estimation of the fault parameters prior to the fault isolation. This is achieved by assuming that the fault instant is a realization of a discrete uniform RV and the fault magnitude is a realization of a *gamma* RV.

1.1. Notation

Let $\mathbb{R}$ and $\mathbb{Z}_+$ denote the set of real numbers and the set of non-negative integers, respectively. The vector $\mathbf{a}_k \in \mathbb{R}^n$, for some $n \in \mathbb{Z}_+$, is a quantity referring to the discrete-time instant t_k. Infinite sequences are denoted as $\{\mathbf{a}_k\} \triangleq \mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2, \ldots$, while finite sequences are denoted as $\mathbf{a}_{k_1:k_2}$, $\forall k_1, k_2 \in \mathbb{Z}_+, k_1 < k_2$. $\mathbf{e}_j \in \mathbb{R}^n$, for some $n \in \mathbb{Z}_+$, is a vector whose jth component is 1 and the others are 0. The function $1_{k,k_f}$ equals 1, $\forall k \geq k_f$ and equals 0 otherwise. $\delta(k)$ denotes the Dirac delta function. $U([l_1, l_2])$ denotes a discrete uniform distribution whose support is the interval $[l_1, l_2] \subset \mathbb{Z}_+$, while $G(\alpha, \beta)$ denotes a *gamma* distribution with shape parameter α and scale parameter β. The $n \times m$ zero matrix is represented by $\mathbf{0}_{n\times m}$, while the n-dimensional identity matrix is denoted by $\mathbf{I}_n$.

2. Problem statement

The present section formally defines the fault diagnosis problem studied here, makes some preliminary assumptions and provides a scheme for implementing the method.

2.1. System state-space model

Let the plant be described by the following discrete-time linear Gaussian state-space model:

$$\mathbf{x}_{k+1} = \mathbf{A}_k\mathbf{x}_k + \mathbf{B}_k\mathbf{u}_k + \boldsymbol{\Gamma}_k\mathbf{w}_k + \boldsymbol{\Xi}_k\mathbf{f}_k \quad (1)$$

$$\mathbf{y}_k = \mathbf{C}_k\mathbf{x}_k + \mathbf{v}_k + \boldsymbol{\Theta}_k\mathbf{f}_k \quad (2)$$

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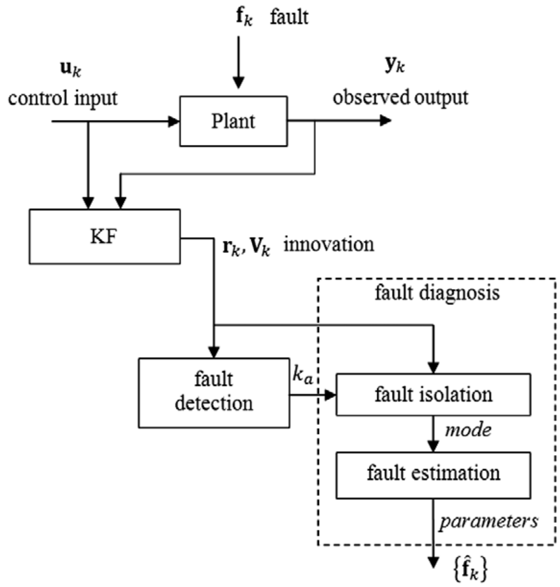

Fig. 1. Block diagram of the overall fault diagnosis scheme.

where $\mathbf{x}_k \in \mathbb{R}^{n_x}$ is the state vector; $\mathbf{y}_k \in \mathbb{R}^{n_y}$ is the vector of observed outputs; $\mathbf{u}_k \in \mathbb{R}^{n_u}$ is the vector of control inputs; $\mathbf{f}_k \in \mathbb{R}^{n_f}$ is the fault vector; $\mathbf{A}_k$, $\mathbf{B}_k$, $\boldsymbol{\Gamma}_k$, $\mathbf{C}_k$, $\boldsymbol{\Xi}_k$, and $\boldsymbol{\Theta}_k$ are known deterministic matrices with appropriate dimensions; $\{\mathbf{w}_k\}$ and $\{\mathbf{v}_k\}$ are mutually independent, zero mean, white, Gaussian sequences with known covariances, $\mathbf{Q}_k$ and $\mathbf{R}_k$, respectively. These sequences are also assumed to be statistically independent of the initial state $\mathbf{x}_0$, which is assumed to be a Gaussian RV with known mean and covariance.

Remark 1. In Section 3, the sequence of fault vectors $\{\mathbf{f}_k\}$ will be considered as an unknown *realization* of a sequence of random variables (RV) statistically independent of $\mathbf{x}_0$, $\{\mathbf{w}_k\}$ and $\{\mathbf{v}_k\}$.

Remark 2. Let the additive fault on the control input vector be denoted by $\delta\mathbf{u}_k$. Such an actuator fault is appropriately described by the fault vector, $\mathbf{f}_k$, in (1)–(2), sufficing to do $\boldsymbol{\Xi}_k\mathbf{f}_k = \mathbf{B}_k\delta\mathbf{u}_k$.

2.2. Scheme for fault diagnosis implementation

Fig. 1 shows the overall scheme adopted for fault diagnosing. The KF designed with the model of the plant under fault-free conditions is used for generating the residual signal, $\mathbf{r}_k$ (with covariance $\mathbf{V}_k$), that is used as data for both fault detection and fault diagnosis. The fault detection module has the function of alarming (e.g., by means of a χ^2 test) the occurrence of some fault. Such binary information, available at the alarm instant k_a, is used for activating the fault diagnosis module, whose function is to infer the location, the form, the instant of occurrence and the magnitude of the fault.

The scheme of Fig. 1 follows the general structure of a system for model-based fault diagnosis (Frank, 1990), which contains two main modules: a *residual generator* (KF), and a *decision maker* (fault detection, fault isolation, and fault estimation). The KF is a convenient residual generator for linear systems subject to additive faults, once the statistical properties of its innovation sequence let one poses the diagnosis problem as commonly found in detection theory applications (Kay, 1998). This will become evident in Section 2.3.

2.3. Residue generation

Under fault-free conditions, i.e. $\mathbf{f}_k = 0$, $\forall k \in \mathbb{Z}_+$, it is well known that the Minimum Mean-Squared Error (MMSE) estimate of $\mathbf{x}_{k+1}$ given the measurements $\mathbf{y}_{0:k}$, as well as the corresponding error covariance are given by the KF algorithm (Anderson & Moore, 1979),

$$\hat{\mathbf{x}}_{k+1|k} = \bar{\mathbf{A}}_k\hat{\mathbf{x}}_{k|k-1} + \mathbf{B}_k\mathbf{u}_k + \mathbf{A}_k\mathbf{K}_k\mathbf{y}_k \tag{3}$$

$$\mathbf{P}_{k+1|k} = \bar{\mathbf{A}}_k\mathbf{P}_{k|k-1}\mathbf{A}'_k + \boldsymbol{\Gamma}_k\mathbf{Q}_k\boldsymbol{\Gamma}'_k \tag{4}$$

where $\bar{\mathbf{A}}_k \triangleq \mathbf{A}_k\left(\mathbf{I}_{n_x} - \mathbf{K}_k\mathbf{C}_k\right)$, and the Kalman gain is $\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{C}'_k\left(\mathbf{C}_k\mathbf{P}_{k|k-1}\mathbf{C}'_k + \mathbf{R}_k\right)^{-1}$. The quantity $\mathbf{r}_k \triangleq \mathbf{y}_k - \hat{\mathbf{y}}_{k|k-1}$, where $\hat{\mathbf{y}}_{k|k-1} \triangleq \mathbf{C}_k\hat{\mathbf{x}}_{k|k-1}$ is the innovation vector, which is a Gaussian RV with zero mean and covariance $\mathbf{V}_k = \mathbf{C}_k\mathbf{P}_{k|k-1}\mathbf{C}'_k + \mathbf{R}_k$. Moreover, the sequence $\{\mathbf{r}_k\}$ can be shown to be white (Anderson & Moore, 1979). The KF innovation sequence $\{\mathbf{r}_k\}$ is frequently used as the residual signal for fault detection and diagnosis (Deshpande, Patwardhan, & Narasimhan, 2009; Gertler, 1988; Prakash, Narasimhan, & Patwardhan, 2005; Prakash, Patwardhan, & Narasimhan, 2002; Willsky & Jones, 1976). The fault diagnosis method proposed in this work is also based on $\{\mathbf{r}_k\}$.

Once an additive fault occurs, the innovation sequence of the KF (3)–(4) will be still white and Gaussian with covariance $\mathbf{V}_k$, but its mean deviates from zero in accordance with a linear function of the sequence of fault vectors, $\{\mathbf{f}_k\}$. A recursive formula for computing the signature of some hypothesized fault on the innovation sequence of the above KF is provided by the following lemma.

Lemma 1. *Let the plant be described by (1)–(2), whose KF, under fault-free conditions, is given by (3)–(4). If $\{\mathbf{f}_k\}$ is some realization of the fault and $k_f \in \mathbb{Z}_+$ is the fault instant, defined as being the largest integer such that $\mathbf{f}_k = \mathbf{0}$, $\forall k < k_f$, then the innovation vector $\mathbf{r}_k$ at any instant k is given by*

$$\mathbf{r}_k = \mathbf{r}_k^N + \mathbf{g}_k \tag{5}$$

where $\left\{\mathbf{r}_k^N\right\}$ is white, Gaussian, with zero mean and covariance $\mathbf{V}_k$, and the fault signature vector is

$$\mathbf{g}_k = \mathbf{C}_k\tilde{\mathbf{g}}_k + \boldsymbol{\Theta}_k\mathbf{f}_k \tag{6}$$

where $\tilde{\mathbf{g}}_k$ is computed recursively by

$$\tilde{\mathbf{g}}_k = \bar{\mathbf{A}}_{k-1}\tilde{\mathbf{g}}_{k-1} + \bar{\mathbf{B}}_{k-1}\mathbf{f}_{k-1} \tag{7}$$

with initial condition $\tilde{\mathbf{g}}_{k_f} = \mathbf{0}$. $\bar{\mathbf{A}}_{k-1}$ was defined after Eq. (4), and $\bar{\mathbf{B}}_{k-1} \triangleq (\boldsymbol{\Xi}_{k-1} - \mathbf{A}_{k-1}\mathbf{K}_{k-1}\boldsymbol{\Theta}_{k-1})$.

Proof. Let $\mathbf{f}_k \neq \mathbf{0}$ for some $k > k_f$. By using Eq. (2), the innovation of the KF (3)–(4) at k can be rewritten as

$$\mathbf{r}_k = \mathbf{C}_k\tilde{\mathbf{x}}_{k|k-1} + \mathbf{v}_k + \boldsymbol{\Theta}_k\mathbf{f}_k \tag{8}$$

where $\tilde{\mathbf{x}}_{k|k-1} \triangleq \mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1}$ is the estimation error, whose mean value depends on the fault vectors between k_f and $k-1$. By taking Eqs. (1)–(3) into account, the estimation error $\tilde{\mathbf{x}}_{k|k-1}$ can be expressed by

$$\tilde{\mathbf{x}}_{k|k-1} = \bar{\mathbf{A}}_{k-1}\tilde{\mathbf{x}}_{k-1|k-2} + \bar{\mathbf{B}}_{k-1}\mathbf{f}_{k-1} + \boldsymbol{\eta}_{k-1} \tag{9}$$

where $\bar{\mathbf{A}}_{k-1}$ and $\bar{\mathbf{B}}_{k-1}$ were previously defined, and $\boldsymbol{\eta}_{k-1} \triangleq \boldsymbol{\Gamma}_{k-1}\mathbf{w}_{k-1} - \mathbf{A}_{k-1}\mathbf{K}_{k-1}\mathbf{v}_{k-1}$. Now define $\tilde{\mathbf{g}}_k \triangleq E\left\{\tilde{\mathbf{x}}_{k|k-1}\right\}$. Therefore, Eq. (7) is obtained by taking the expectation of (9). Finally, define $\mathbf{g}_k \triangleq E\{\mathbf{r}_k\}$. Thus by taking the expectation of (8), the desired signature vector, Eq. (6), is obtained. □

The ability to predict the fault signature on the residual signal is the crucial element to the fault diagnosis problem, as will be seen in what follows.

2.4. Fault diagnosis problem

The fault diagnosis problem is formulated by considering as data a set of innovation vectors referring to a fixed-width time window with initial instant k_a.