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Interpolation for gain-scheduled control with guarantees[☆]

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ABSTRACT

Here, a methodology is presented which considers the interpolation of linear time-invariant (LTI) controllers designed for different operating points of a nonlinear system in order to produce a gain-scheduled controller. Guarantees of closed-loop quadratic stability and performance at intermediate interpolation points are presented in terms of a set of linear matrix inequalities (LMIs). The proposed interpolation scheme can be applied in cases where the system must remain at the operating points most of the time and the transitions from one point to another rarely occur, *e.g.*, chemical processes, satellites.

1. Introduction

Gain scheduling has been used successfully to control nonlinear systems for many decades and in many different applications, such as autopilots and chemical processes (Rugh & Shamma, 2000). It consists in selecting a family of operating points, or more generally regions, where the system can be described by a linear model. A linear controller is designed for each region which should guarantee performance and robustness in that region. Finally, the controllers are changed according to a physical parameter measured in real time, which detects in what region the system is working at each time. The change of controllers can be implemented either gradually by interpolation of certain parameters or by switching.

In practice, switching among controllers may create instability of the closed-loop system (Liberzon, 2003). Unstable modes and degraded performance may come from the transition dynamics, which are not contained in the information provided by each linear model. Usually, a way to mitigate this problem is to impose a certain dwell time (Hespanha & Morse, 1999). However, this is not able to prevent the undesirable transients, which may require complex algorithms to reduce their negative effects.

On the other hand, interpolation provides smooth changes between controllers. In general, this is a fairly simple solution in cases of SISO problems or fixed structure controllers, such as PIDs or lateral-directional aircraft control, due to the fact that only certain fixed parameters are interpolated, *e.g.* gains, poles, and numerator/denominator coefficients. However, in more general cases where the sets of controllers have been designed independently or are MIMO models, the implementation of parameter interpolation is not as simple. In addition, in these cases it is convenient to interpolate the controller state-space realization instead of parameters from its transfer matrix.

Stability and performance guarantees in the whole operating envelope can be obtained using linear parameter varying (LPV) systems theory (Apkarian, Gahinet, & Becker, 1995; Wu, Yang, Packard, & Becker, 1996). The main problem of this method is the computational effort needed to obtain an LPV controller which limits its use to low-order and medium-order systems. In addition, in many fields, *e.g.* aerospace, there is a strong interest of practitioners in using the gain-scheduling method, based on optimized designs at different operating points.

For controllers designed independently for each point, previous results have focused on stability (Chang & Rasmussen, 2008; Stilwell & Rugh, 2000) or on controller switching instead (Blanchini, Miani, & Mesquine, 2009; Hespanha & Morse, 2002). In particular, in Chang and Rasmussen (2008), Youla parameterization has been used, but a network of controllers is produced which

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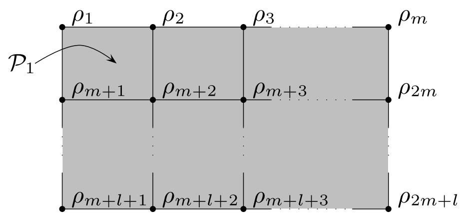

Fig. 1. Example of division of the region $\mathcal{P}$.

significantly increases the order of the resulting gain-scheduled control. Some recent results consider the performance problems by establishing an adequate controller initial condition when switching (Hespanha, Santesco, & Stewart, 2007) or by injecting stabilizing signals among the local controllers, based on bumpless and antiwindup transfer compensators (Hencey & Alleyne, 2009). There are no results that have focused on both stability and performance, based on the adequate selection of the state-space realizations for interpolation.

This paper focuses on formulating a stability-preserving interpolation scheme with a performance level guarantee in the state-space framework. The aim is to obtain gain-scheduled controllers with similar stability properties as LPV versions and with the possibility of tuning each linear time-invariant (LTI) controller independently. The next section presents the problem statement and Section 3 gives the main results, illustrated by a short example in Section 4. The paper ends in Section 5 with some concluding remarks.

2. Problem statement

Consider the set of linear models

$$G_i(s) = \left[\begin{array}{c|cc} A_i & B_{1,i} & B_2 \\ \hline C_{1,i} & D_{11,i} & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right], \quad i \in \mathbb{I}_{n_p} \tag{1}$$

describing the local dynamic behavior of a nonlinear or time-varying system at each operating point parameterized by $\rho_i \in \mathcal{P}$, with $A_i \in \mathbb{R}^{n\times n}$ and $\mathbb{I}_{n_p} = \{1, \dots, n_p\}$. The set of points $\{\rho_1, \dots, \rho_{n_p}\}$ divides the region $\mathcal{P}$ into a set of subregions $\mathcal{P}_j$ defined by the vertices $\mathcal{V}_j \subseteq \{\rho_1, \dots, \rho_{n_p}\}$, as illustrated in Fig. 1. Then, any point $\rho \in \mathcal{P}_j$ can be expressed as a convex combination of the vertices $\mathcal{V}_j$, *i.e.*,

$$\rho = \sum_{i=1}^{n_p} \alpha_i \rho_i \tag{2}$$

where $\alpha_1 + \dots + \alpha_{n_p} = 1$ and $\alpha_i \geq 0$, $\forall \rho_i \in \mathcal{V}_j, \alpha_i = 0$, $\forall \rho_i \notin \mathcal{V}_j$.

The local dynamics at any point $\rho \in \mathcal{P}_j$ is assumed to be described as a linear combination of the state-space realizations corresponding to the vertices $\mathcal{V}_j$:

$$G(\rho) : \begin{cases} \dot{x} = A(\rho)x + B_1(\rho)w + B_2 u, \\ z = C_1(\rho)x + D_{11}(\rho)w + D_{12}u, \\ y = C_2 x + D_{21} w, \end{cases} \tag{3}$$

where

$$\begin{bmatrix} A(\rho) & B_1(\rho) \\ C_1(\rho) & D_{11}(\rho) \end{bmatrix} = \sum_{i=1}^{n_p} \alpha_i(\rho) \begin{bmatrix} A_i & B_{1,i} \\ C_{1,i} & D_{11,i} \end{bmatrix}$$

and $\alpha_i(\rho)$ is the coordinate corresponding to ρ_i.

According to (2), only the matrices corresponding to $\rho_i \in \mathcal{V}_j$ are needed to compute system (3). This class of models is called *piecewise affine* LPV systems (Lim & How, 2003); it includes the classical affine LPV models. The assumption that B_2, C_2, D_{12}, and D_{21} are constant does not impose any serious constraints, and can be fulfilled by simply filtering the input u and/or the output y (see Apkarian et al., 1995).

It is assumed that there exists a stabilizing linear controller designed beforehand and independently for each plant $G_i(s)$:

$$K_i(s) = \left[\begin{array}{c|c} A_{k,i} & B_{k,i} \\ \hline C_{k,i} & D_{k,i} \end{array}\right], \quad i = 1, \dots, n_p, \tag{4}$$

which achieves certain performance specifications, with $A_{k,i} \in \mathbb{R}^{n_c\times n_c}$. This differs from other synthesis procedures applicable to the plant (3) such as the gridding method proposed by Wu et al. (1996) or the switching LPV framework of Lim and How (2003), where the local controllers are computed simultaneously.

Then, the objective is to formulate an interpolation scheme for the state-space realizations (4) such that the gain-scheduled controller

$$K(\rho) : \begin{cases} \dot{x}_k = A_k(\rho)x_k + B_k(\rho)y, \\ u = C_k(\rho)x_k + D_k(\rho)y \end{cases} \tag{5}$$

stabilizes the plant $G(\rho)$ defined in (3) at any point $\rho \in \mathcal{P}$, with $A_k(\rho) \in \mathbb{R}^{n_k\times n_k}$. Note that the order of the local controllers (4) may differ from the order of the gain-scheduled controller (5) (*i.e.*, in general, $n_c \neq n_k$).

3. Main results

The following lemma provides a systematic method to find a quadratically stable interpolation of several Hurwitz matrices. If the set of matrices A_i represents the local dynamics of an LPV system at the vertices of a convex hull $\text{co}\{\rho_1, \dots, \rho_{n_p}\}$, the following result states that, given a set of Hurwitz matrices, it is always possible to construct a quadratically stable affine LPV matrix.

Lemma 3.1. *Given a set of matrices A_i associated to each vertex of the convex hull $\Theta = \text{co}\{\rho_1, \dots, \rho_{n_p}\}$, the following statements are equivalent.*

(i) *A_i is Hurwitz for all $i \in \mathbb{I}_{n_p}$,*
(ii) *there exist n_p matrix transformations T_i such that the LPV matrix*

$$\tilde{A}(\rho) = \sum_{i=1}^{n_p} \alpha_i(\rho)\tilde{A}_i = \sum_{i=1}^{n_p} \alpha_i(\rho) T_i A_i T_i^{-1} \tag{6}$$

is quadratically stable for all $\rho \in \Theta$, with $\alpha_i(\rho) = \alpha_i$ in $\rho = \sum_{i=1}^{n_p} \alpha_i\rho_i$ such that $\sum_{i=1}^{n_p} \alpha_i = 1$.

Proof. (i) $\Rightarrow$ (ii). If A_i is Hurwitz, then $\exists X_i > 0$ such that $X_iA_i + A_i^TX_i < 0$, $i \in \mathbb{I}_{n_p}$. According to Hespanha and Morse (2002), it is always possible to find state transformations T_i (*e.g.* $T_i = X_i^{1/2}$) such that

$$X\tilde{A}_i + \tilde{A}_i^T X < 0, \quad \forall i \in \mathbb{I}_{n_p} \tag{7}$$

for a common $X > 0$, with $\tilde{A}_i = T_iA_iT_i^{-1}$. Finding the coordinates $\alpha_i(\rho)$, with ρ as a convex combination of the vertices of Θ, the LPV matrix (6) can be constructed. Based on $\alpha_i \geq 0$, $\forall i \in \mathbb{I}_{n_p}$, inequalities (7) and linearity,

$$X\left(\sum_{i=1}^{n_p} \alpha_i(\rho)\tilde{A}_i\right) + \left(\sum_{i=1}^{n_p} \alpha_i(\rho)\tilde{A}_i\right)^T X < 0, \tag{8}$$

and thus the quadratical stability of $\tilde{A}(\rho)$ is proved.

(ii) $\Rightarrow$ (i). Take $\rho = \rho_m$, with ρ_m one of the vertices of Θ; then $\alpha_m = 1$, and $\alpha_i = 0$, $\forall i \neq m$. Therefore, $\tilde{A}(\rho) = \tilde{A}_m$, and from (8) it can be concluded that $\tilde{A}_m$ is Hurwitz, and thus A_m. $\square$