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Stationary consensus of heterogeneous multi-agent systems with bounded communication delays☆

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ABSTRACT

Consensus seeking is investigated for the discrete-time heterogeneous multi-agent systems composed of first-order agents and second-order agents, and two stationary consensus algorithms are constructed for the first-order agents and the second-order agents, respectively. Based on the properties of nonnegative matrices, sufficient consensus criteria are obtained for the agents with bounded communication delays under fixed topology and switching topologies, respectively. With some prerequisites on the coupling weights and the sampling interval, the asymptotic consensus achievement of the dynamic agents is independent of the communication delay, but strictly depends on the connectedness of the interconnection topology. Simulation results illustrate the correctness of the results.

1. Introduction

As one of the most typical collective behaviors of multi-agent systems, consensus, which means that the outputs of several spatially distributed agents reach a common value, has attracted more and more attention in recent years from various research communities, such as physics, artificial intelligence, automatic control, etc.

Up to now, based on different analysis methods, such as the method based on the properties of nonnegative matrices (Jadbabaie, Lin, & Morse, 2003; Lin & Jia, 2009; Ren & Beard, 2005; Xiao & Wang, 2006), the Lyapunov functions (Hong, Gao, Cheng, & Jiang, 2007; Ni & Cheng, 2010), the frequency-domain analysis (Lee & Spong, 2006; Tian & Liu, 2008; Yang, Bertozzi, & Wang, 2008), etc., many consensus algorithms have been proposed and consensus criteria have been obtained for the first-order and the second-order multi-agent systems, respectively, under fixed topology, switching topology or time delays. However, numerous existing results on the consensus problem are mostly given for the homogeneous multi-agent systems, in which all the agents have the identical dynamics. In real engineering application, the dynamics of the agents coupled with each other are always different for various restrictions, but there has been little attention paid on the consensus problem of the heterogeneous multi-agent systems, which consist of the agents with different dynamics. Lee and Spong (2006) studied the consensus seeking of heterogeneous multiple continuous-time dynamic agents, whose dynamics are described as strictly stable linear systems, with non-uniform communication delays, and the decentralized delay-independent consensus conditions are obtained according to the frequency-domain analysis and the spectral radius theorem.

The objective of this paper is to extend the method based on combining the digraphs with the nonnegative matrices (Jadbabaie et al., 2003; Lin & Jia, 2009; Ren & Beard, 2005; Xiao & Wang, 2006) to the discrete-time heterogeneous multi-agent systems composed of first-order agents and second-order agents. To solve the consensus problem of the heterogeneous multi-agent systems, the stationary consensus algorithms are proposed for the first-order agents and second-order agents, respectively. Sufficient consensus conditions with some prerequisites are obtained for the multi-agent systems with bounded communication delays under fixed topology and switching topologies, respectively, by reformulating the delayed system as an equivalent undelayed system.

2. Preliminaries

A weighted digraph $G = (V, E, A)$ of order n consists of a set of vertices $V = \{1, \ldots, n\}$, a set of edges $E \subseteq V \times V$ and a weighted adjacency matrix $A = [a_{ij}] \in R^{n\times n}$ with $a_{ij} \geq 0$. A directed edge from i to j in G is denoted by $e_{ij} = (i, j) \in E$, which means that

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the node j can obtain information from the node i. Assume $a_{ji} > 0 \Leftrightarrow e_{ij} \in E$ and $a_{ii} = 0$ for all $i \in \{1, \ldots, n\}$. The set of neighbors of node i is denoted by $N_i = \{j \in V : (j, i) \in E\}$. The Laplacian matrix of the digraph G is defined as $L = D - A = [l_{ij}] \in R^{n\times n}$, where $D = \text{diag}\{\sum_{j=1}^{n} a_{ij}, i = 1, \ldots, n\}$ is the degree matrix. In the digraph G, a directed path from node i_1 to node i_s is a sequence of ordered edges of the form $(i_1, i_2), \ldots, (i_{s-1}, i_s)$ where $i_j \in V$. A digraph is said to have a spanning tree, if there exists a node such that there is a directed path from this node to every other node.

A matrix $C = [c_{ij}] \in R^{n\times r}$ is nonnegative if all its elements c_{ij} are nonnegative. If a nonnegative matrix $C \in R^{n\times r}$ satisfies $C1_r = 1_n$, then it is said to be (row) stochastic. A stochastic matrix $B \in R^{n\times n}$ is said to be indecomposable and aperiodic (*SIA*) if $\lim_{m\to\infty} B^m = 1_n f^T$ where $f \in R^n$. In this paper, $1_n = [1, 1, \ldots, 1]^T$, and I_n denotes a $n \times n$ identity matrix.

Lemma 1 (*Wolfowitz, 1963*). *Let $P_1, P_2, \ldots, P_k \in R^{n\times n}$ be a finite set of SIA matrices with the property that for each sequence $P_{i_1}, P_{i_2}, \ldots, P_{i_j}$ with positive length, the matrix product $P_{i_j}P_{i_{j-1}}\cdots P_{i_1}$ is SIA. Then, for each infinite sequence $P_{i_1}, P_{i_2}, \ldots,$ there exists a vector $f \in R^n$ such that $\lim_{j\to\infty} P_{i_j}P_{i_{j-1}}\cdots P_{i_1} = 1_n f^T$.*

3. Heterogeneous multi-agent systems

To describe the heterogeneous multi-agent systems composed of first-order and second-order agents clearly, the first m ($m < n$) agents are assumed to be second-order agents, while the rest $(n - m)$ agents are first-order agents. The second-order agents are given by

$$x_i(k+1) = x_i(k) + T v_i(k) + \frac{T^2}{2} u_i(k),$$
$$v_i(k+1) = v_i(k) + T u_i(k), \quad i = 1, \ldots, m, \tag{1}$$

where $x_i \in R$, $v_i \in R$ and $u_i \in R$ are the position, velocity and control input, respectively, of agent i. Besides, the first-order agents are described as

$$x_l(k+1) = x_l(k) + T u_l(k), \quad l = m+1, \ldots, n, \tag{2}$$

where $x_l \in R$ and $u_l \in R$ are the state and control input of agent l respectively. In (1) and (2), $T > 0$ is the sampling interval.

In a multi-agent system, each agent can be considered as a node, and information flow between neighboring agents can be regarded as a directed edge. Hence, the interconnection topology of the agents (1) and (2) is usually described as a digraph $G = (V, E, A)$.

Remark 1. (1) and (2) are the sampled-data models with zero-order hold of the continuous-time second-order multi-agent systems (Hong et al., 2007; Ren & Atkins, 2007) and first-order multi-agent systems (Olfati-Saber & Murray, 2004) respectively. Up to now, consensus problem of the first-order multi-agent systems (2) has been extensively studied, but only a little attention has been paid on the consensus of second-order multi-agent systems (1) (Hayakawa, Matsuzawa, & Hara, 2006; Liu & Liu, 2009).

Then, we say that the agents (1) and (2) asymptotically converge to a *stationary consensus*, if $\lim_{k\to\infty} x_i(k) = c, i = 1, \ldots, n$, where c is a constant. To solve the stationary consensus problem of second-order agents (1), a general stationary consensus algorithm (Hong et al., 2007; Ren & Atkins, 2007) is proposed as follows

$$u_i(k) = -\kappa v_i(k) + \sum_{j\in N_i(k)} a_{ij}(k)(x_j(k - \tau_{ij}(k)) - x_i(k)), \tag{3}$$

where $\kappa > 0$ is control parameter, the communication delay $0 \le \tau_{ij}(k) \le \tau_{\max}$ corresponds to the information flow from agent j to agent i, $a_{ij}(k) > 0, j \in N_i(k) = N_i^s(k) \cup N_i^{sf}(k)$ is the coupling weight chosen from any finite set, $N_i^s(k)$ and $N_i^{sf}(k)$ are the agent i's second-order and first-order neighboring agents respectively. Define $z_i(k) = x_i(k) + T v_i(k)$, $i = 1, \ldots, m$, and the closed form of the second-order agents (1) with the algorithm (3) is

$$\begin{aligned} x_i(k+1) &= \frac{T\kappa}{2} x_i(k) + \frac{2 - T\kappa}{2} z_i(k) \\ &\quad + \frac{T^2}{2}\left(\sum_{j\in N_i(k)} a_{ij}(k)(x_j(k - \tau_{ij}(k)) - x_i(k))\right), \\ z_i(k+1) &= \frac{4 - 3T\kappa}{2} z_i(k) + \frac{3T\kappa - 2}{2} x_i(k) \\ &\quad + \frac{3T^2}{2}\left(\sum_{j\in N_i(k)} a_{ij}(k)(x_j(k - \tau_{ij}(k)) - x_i(k))\right), \\ &\quad i = 1, \ldots, m. \end{aligned} \tag{4}$$

For the first-order agents, we adopt the following consensus algorithm

$$u_l(k) = \sum_{j\in N_l(k)} a_{lj}(k)(x_j(k - \tau_{lj}(k)) - x_l(k)), \tag{5}$$

where $a_{lj}(k) > 0, j \in N_l(k) = N_l^f(k) \cup N_l^{fs}(k)$ is the coupling weight chosen from any finite set, $0 \le \tau_{lj}(k) \le \tau_{\max}$ is the communication delay form agent j to agent l, $N_l^f(k)$ and $N_l^{fs}(k)$ are the agent l's first-order and second-order neighboring agents respectively. In (3) and (5), $\tau_{\max} = \max_{i,j}\{\tau_{ij}(k)\}$. With (5), the closed form of the first-order agents (2) is

$$x_l(k+1) = x_l(k) + T\left(\sum_{j\in N_l(k)} a_{lj}(k)(x_j(k - \tau_{lj}(k)) - x_l(k))\right),$$
$$l = m+1, \ldots, n. \tag{6}$$

Remark 2. Stationary consensus of the continuous-time heterogeneous multi-agent systems with time-invariant communication delays under fixed topology has been analyzed in Lee and Spong (2006), and the frequency-domain consensus conditions have been obtained based on the Gerschgorin disk theorem. By using the properties of the nonnegative matrices (Jadbabaie et al., 2003; Lin & Jia, 2009; Ren & Beard, 2005; Wolfowitz, 1963; Xiao & Wang, 2006), we will obtain the consensus conditions of (4) and (6) with switching topologies and bounded communication delays for the first time in this paper.

Defining $w(k) = [x_s^T(k), z_s^T(k), x_f^T(k)]^T$, in which $x_s = [x_1, \ldots, x_m]^T$, $z_s = [z_1, \ldots, z_m]^T$ and $x_f = [x_{m+1}, \ldots, x_n]^T$, we rewrite the systems (4) and (6) as

$$w(k+1) = \Xi_0(k) w(k) + \sum_{p=1}^{\tau_{\max}} \Xi_p(k) w(k-p), \tag{7}$$

where $\Xi_p(k)$, $p = 0, 1, \ldots, \tau_{\max}$ are $(n+m)\times(n+m)$ matrices satisfying $\sum_{p=0}^{\tau_{\max}} \Xi_p(k) = \Xi(k)$, and $\Xi(k)$ is given by

$$\Xi(k) = \begin{bmatrix} \frac{T\kappa}{2} I_m - \frac{T^2}{2}\bar{L}_s(k) & \frac{2 - T\kappa}{2} I_m & \frac{T^2}{2} A_{sf}(k) \\ \frac{3T\kappa - 2}{2} I_m - \frac{3T^2}{2}\bar{L}_s(k) & \frac{4 - 3T\kappa}{2} I_m & \frac{3T^2}{2} A_{sf}(k) \\ T A_{fs}(k) & 0 & I_{n-m} - T\bar{L}_f(k) \end{bmatrix}, \tag{8}$$

where $\bar{L}_s(k) = L_s(k) + D_{sf}(k)$ and $\bar{L}_f(k) = L_f(k) + D_{fs}(k)$, $L_s(k)$ and $L_f(k)$ are the Laplacian matrices of the digraphs composed of the m second-order agents and the $(n - m)$ first-order agents respectively, $D_{sf}(k) = \text{diag}\{\sum_{j\in N_i^{sf}} a_{ij}(k),\ i = 1, \ldots, m\}$, $D_{fs}(k) = \text{diag}\{\sum_{j\in N_l^{fs}} a_{lj}(k),\ l = m+1, \ldots, n\}$, and $A_{sf}(k)$ and $A_{fs}(k)$ satisfy