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Optimal ripple-free deadbeat control using an integral of time squared error (ITSE) index[☆]

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ABSTRACT

This paper describes an optimal ripple-free deadbeat control strategy for single-input–single-output (SISO) linear sampled data plants. The cost function to be minimized is a linear combination of a time-weighted cumulative term that penalizes the tracking error, that is, an integral of time squared error (ITSE) cost term, and a cumulative term which penalizes the control signal deviations from its steady-state value. The optimization problem turns out to be convex, and closed-form solutions are obtained. An example is included to illustrate our results.

1. Introduction

In optimal control, the choice of the performance index is a key issue, since it defines the quality measure used to assess controller performance. Commonly used time-domain indices include the ISE (integral[1] squared error) index, which is related to the 2-norm, the IAE (integral of the absolute error) index, related to the 1-norm, and the ITSE (integral of time squared error) index (Duarte-Mermoud & Prieto, 2004; Ogata, 1997).

The ITSE index has been used in connection with the tuning of proportional–integral–derivative (PID) controllers (Ogata, 1997). A deeper treatment of this index can be found in Carrasco and Salgado (2009), where a set of analytical tools for the optimal design of controllers for stable discrete-time single-input–single-output (SISO) plants is presented. Also, Barbargires and Karybakas (1994) presents optimal deadbeat controller design for discrete-time SISO plants based on a time-weighted performance index. However, ripple-free behaviour is not considered in that work.

This article deals with the design of controllers that, for step references, achieve ripple-free deadbeat (RFDB) control for sampled-data plants and, at the same time, minimize the time-weighted cost function

$$J = \lambda J_e + (1-\lambda) J_u, \qquad (1)$$

where $\lambda \in [0; 1]$ is a weighting parameter, and

$$J_e = \sum_{k=0}^{\infty} k e(k)^2, \qquad J_u = \sum_{k=0}^{\infty} (u(k) - u_{ss})^2. \qquad (2)$$

In (2), $e(k)$ denotes the tracking error, $u(k)$ the control input, and u_{ss} the steady-state value of $u(k)$. Due to the time weighting of $e(k)$, we have that, for J_e to become small, the tracking error must converge to zero rapidly. This property suggests that the unconstrained optimization of J_e will have a negative impact on the control effort. The presence of J_u allows one to deal with that problem by choosing an appropriate value for λ.

The main contribution of this article is finding the optimal RFDB controller that minimizes the index J in (1). A numerical example is provided to illustrate our results.

2. Ripple-free deadbeat control

Deadbeat is a well-known technique for the design of discrete-time control systems, whose purpose is to perfectly track a reference in the minimum number of sampling periods (see, e.g., Emami-Naein & Franklin, 1982; Goodwin, Graebe, & Salgado, 2001; Salgado, Oyarzún, & Silva, 2007). The number of sampling

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[1] In this paper we work in discrete-time. Therefore integration must be interpreted as a cumulative sum.

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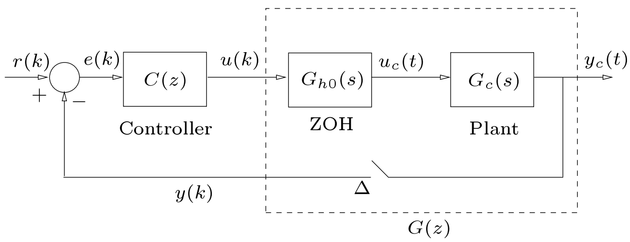

Fig. 1. Standard one-degree-of-freedom sampled-data control loop.

times required for the tracking error to converge to zero is known as the *deadbeat horizon*. In the context of sampled-data systems, deadbeat control should avoid intersample ripple in the continuous-time output of the plant. This requirement has motivated the development of techniques for designing RFDB controllers (see, e.g., Casavola, Mosca, & Zecca, 1999; Nobuyama, 1993; Paz, 1999). The optimal design of RFDB controllers has been already addressed using l_∞, l_1, and $\mathcal{H}_2$ norms (see, e.g., Casavola et al., 1999; Salgado & Oyarzún, 2007; Salgado et al., 2007).

Fig. 1 shows the sampled-data control loop considered in this paper. The goal is to track a step reference $r(k) = \upsilon\mu(k)$, achieving $y_c(t) = \upsilon, \forall t \geqslant \eta\Delta$, and a control signal such that $u(k) = u_{ss}$ (a constant), $\forall k \geqslant \eta$. Here, $\eta \in \mathbb{N}_0$ is the deadbeat horizon. For simplicity, $\upsilon = 1$ is assumed. The continuous-time model of the plant is assumed to have the form $G_c(s) = \frac{B_c(s)}{A_c(s)}e^{-s\tau}$, with $\tau \geqslant 0$, where $B_c(s)$ and $A_c(s)$ are coprime polynomials in s. The sampled data representation of $G_c(s)$, using a zero-order hold, is given by $G(z) = \frac{B(z)}{A(z)}$, where $B(z)$ and $A(z)$ are coprime polynomials in z of degrees n and m, respectively, with $n \geqslant m$. Without loss of generality, we assume that $B(1) = 1$. On the other hand, $A(z)$ is factorized as $A(z) = A_+(z)A_-(z)$, where the polynomial $A_+(z)$, of degree n_+, contains all roots of $A(z)$ in $\{z \in \mathbb{C} : |z| \geq 1\}$, while $A_-(z)$, of degree n_-, contains all roots in $\{z \in \mathbb{C} : |z| < 1\}$.

To achieve an RFDB response, it is necessary that the synthesized controller does not cancel the minimum-phase zeros of $G(z)$ (Goodwin et al., 2001; Salgado et al., 2007). The solution to the problem thus requires that the complementary sensitivity $T(z)$ and the control sensitivity $S_u(z)$ are finite impulse response (FIR) transfer functions of order η (Karybakas & Barbargires, 1994; Sirisena, 1985). This ensures that both $e(k)$ and $u(k)$ settle to constant values in the specified deadbeat horizon. Also, by requiring that $T(1) = 1$, we guarantee zero steady-state error. On the other hand, the minimum deadbeat horizon to get RFDB control is given by $\eta_{\min} = n + n_+$ (Nobuyama, 1993), and, hence, any arbitrary horizon can be written as $\eta = \eta_{\min} + \ell$, with $\ell \in \mathbb{N}_0$. When $\ell = 0$, there exists only one controller, say $C_o(z)$, providing an RFDB response. That controller satisfies

$$A(z)L_o(z) + B(z)P_o(z) = z^{\eta_{\min}}A_-(z), \tag{3}$$

where $C_o(z) = P_o(z)/L_o(z)$ is biproper with $L_o(z)$ of order n such that $L_o(1) = 0$. Note from (3) that $P_o(z) = \tilde{P}_o(z)A_-(z)$, with $\tilde{P}_o(1) = 1$. It is known that every stabilizing RFDB controller $C(z)$ can be written as Salgado et al. (2007)

$$C(z) = \frac{P_o(z) + X(z)A(z)}{L_o(z) - X(z)B(z)}, \tag{4}$$

where $X(z)$ is an FIR transfer function of order ℓ such that $X(1) = 0$. Thus, $X(z) = (z-1)\tilde{D}(z)z^{-\ell}$, where $\tilde{D}(z)$ is a polynomial of order $\ell - 1$ or less. The optimal design of an RFDB controller amounts to finding the polynomial $\tilde{D}(z)$ that minimizes J.

3. Optimal designs

This section presents a technique to design optimal controllers using the index defined in (1). By using Parseval's theorem Goodwin et al. (2001), we can write J_e and J_u in (2) as follows:

$$J_e = -\frac{1}{2\pi j}\oint \frac{\mathrm{d}E(z)}{\mathrm{d}z}E(z^{-1})\mathrm{d}z, \tag{5}$$

$$J_u = \frac{1}{2\pi j}\oint \frac{1}{z}F(z)F(z^{-1})\mathrm{d}z, \tag{6}$$

where $E(z) = S(z)R(z)$ is the $\mathcal{Z}$ transform of the tracking error, and $F(z) = S_u(z)R(z) - u_{ss}\frac{z}{z-1}$ is the $\mathcal{Z}$ transform of $u(k) - u_{ss}$. Furthermore, given that $r(k) = \mu(k)$, we can proceed as in Salgado et al. (2007) to write $E(z) = M_e(z) - N_e(z)\tilde{D}(z)$ and $F(z) = M_u(z) - N_u(z)\tilde{D}(z)$, where

$$M_e(z) = \frac{z^{\eta_{\min}} - B(z)\tilde{P}_o(z)}{z^{\eta_{\min}-1}(z-1)} = \sum_{i=0}^{\eta_{\min}-1} m_{e_i}z^{-i}, \tag{7}$$

$$N_e(z) = \frac{B(z)A_+(z)}{z^{\eta-1}} = \sum_{i=0}^{\eta-1} n_{e_i}z^{-i}, \tag{8}$$

$$M_u(z) = \frac{A(z)\tilde{P}_o(z) - A(1)z^{\eta_{\min}}}{z^{\eta_{\min}-1}(z-1)} = \sum_{i=0}^{\eta_{\min}-1} m_{u_i}z^{-i}, \tag{9}$$

$$N_u(z) = -\frac{A(z)A_+(z)}{z^{\eta-1}} = \sum_{i=0}^{\eta-1} n_{u_i}z^{-i}, \tag{10}$$

and m_{e_i}, n_{e_i}, m_{u_i}, and n_{u_i} are real coefficients (note that $\tilde{P}_o(1) = 1$, $B(1) = 1$, and $m \leqslant n$ implies that $M_e(z)$, $N_e(z)$, $M_u(z)$, and $N_u(z)$ are FIR transfer functions). To further simplify the problem formulation, we define

$$\boldsymbol{\zeta}(z) \triangleq \begin{bmatrix} z^0 & \cdots & z^{\ell-1}\end{bmatrix}^T, \qquad \tilde{\mathbf{d}} \triangleq \begin{bmatrix}\tilde{d}_0 & \cdots & \tilde{d}_{\ell-1}\end{bmatrix}^T. \tag{11}$$

Given that $\tilde{D}(z) = \boldsymbol{\zeta}(z)^T\tilde{\mathbf{d}}$, the problem of finding the optimal polynomial $\tilde{D}(z)$ reduces to finding the optimal vector of coefficients $\tilde{\mathbf{d}}$. The costs J_e and J_u can be written as functions of $\tilde{\mathbf{d}}$, as

$$J_e(\tilde{\mathbf{d}}) = T_e - \tilde{\mathbf{d}}^T \cdot \mathbf{K}_e + \tilde{\mathbf{d}}^T \cdot \mathbf{L}_e \cdot \tilde{\mathbf{d}}, \tag{12}$$

$$J_u(\tilde{\mathbf{d}}) = T_u - \tilde{\mathbf{d}}^T \cdot \mathbf{K}_u + \tilde{\mathbf{d}}^T \cdot \mathbf{L}_u \cdot \tilde{\mathbf{d}}, \tag{13}$$

where $T_e, T_u \in \mathbb{R}$, $\mathbf{K}_e, \mathbf{K}_u \in \mathbb{R}^{\ell\times 1}$, and $\mathbf{L}_e, \mathbf{L}_u \in \mathbb{R}^{\ell\times\ell}$ are given by

$$T_e = -\frac{1}{2\pi j}\oint \frac{\mathrm{d}M_e(z)}{\mathrm{d}z}M_e(z^{-1})\mathrm{d}z, \tag{14}$$

$$T_u = \frac{1}{2\pi j}\oint \frac{1}{z}M_u(z)M_u(z^{-1})\mathrm{d}z, \tag{15}$$

$$\mathbf{K}_e = -\frac{1}{2\pi j}\oint \Bigg(M_e(z^{-1})\frac{d\left[N_e(z)\boldsymbol{\zeta}(z)\right]}{\mathrm{d}z} + \frac{\mathrm{d}M_e(z)}{\mathrm{d}z}N_e(z^{-1})\boldsymbol{\zeta}(z^{-1})\Bigg)\mathrm{d}z, \tag{16}$$

$$\mathbf{K}_u = \frac{1}{2\pi j}\oint \frac{1}{z}(M_u(z)N_u(z^{-1})\boldsymbol{\zeta}(z^{-1}) + M_u(z^{-1})N_u(z)\boldsymbol{\zeta}(z))\mathrm{d}z, \tag{17}$$

$$\mathbf{L}_e = -\frac{1}{2\pi j}\oint \frac{d[\boldsymbol{\zeta}(z)N_e(z)]}{\mathrm{d}z}N_e(z^{-1})\boldsymbol{\zeta}(z^{-1})^T\mathrm{d}z, \tag{18}$$

$$\mathbf{L}_u = \frac{1}{2\pi j}\oint \frac{1}{z}\boldsymbol{\zeta}(z)N_u(z)N_u(z^{-1})\boldsymbol{\zeta}(z^{-1})^T\mathrm{d}z. \tag{19}$$